

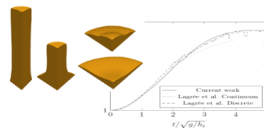
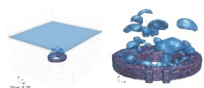
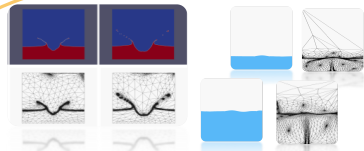
## Massively Parallel Anisotropic Meshing Framework for CFD and Data Sciences

Elie Hachem, Aurélien Larcher, R. Valette

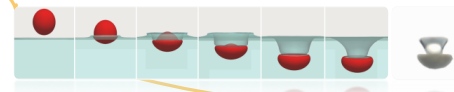
CFL Computing and Fluids Research Group

CEMEF MINES ParisTech | PSL Research University

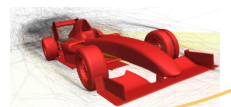
Applied mathematics



Fluid Mechanics



Parallel Computing



- 1- Context and Motivation
- 2- Immersed method and Eulerian Framework
- 2- Anisotropic meshing with conservative interpolation
- 3- Stabilized FEM for complex fluids: turbulent flows, multiphase flows
- 4- Towards coupling CFD and Data Sciences

# MINES ParisTech, PSL

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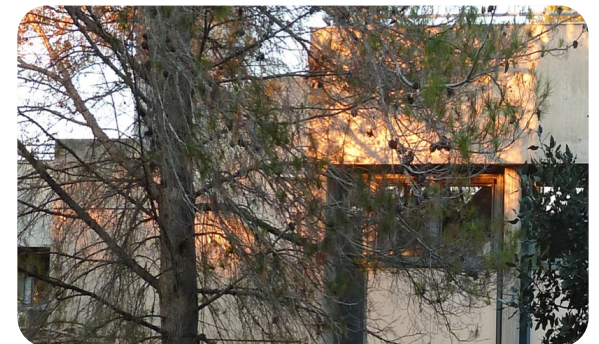
- ▶ Founded in 1783
- ▶ 2300 persons, 240 researchers and professors, 400 PhDs
- ▶ 5 sites : Paris, Evry, Fontainebleau, Palaiseau and **Sophia Antipolis**
- ▶ 18 research centers
- ▶ 5 departments
  - ▶ Geosciences
  - ▶ Mathematics
  - ▶ Mechanics and Materials
  - ▶ Energy and Processes
  - ▶ Economy, management and Society



Paris



Sophia Antipolis



My office

## CFL Computing and Fluids Research group

- ▶ 11 researchers and professors
- ▶ 19 PhDs and 3 Postdocs



### Numerical framework (C++ parallel FEM library)

#### Applied Mathematics

Unsteady Navier-Stokes equations  
Fluid-Structure interaction  
Turbulence and heat transfer  
Adjoint solution and control

#### Fluid Mechanics

Multiphase flows (liquid, vapor, solid...)  
Yield stress and granular flows  
Darcy and Porous media  
Interface and surface tension

#### High Performance Computing

Scalable implicit massively parallel solver  
A posteriori error estimator  
Anisotropic mesh adaptation



# Challenges

« ...essentially driven by real industrial applications »

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*Industrial Furnaces*



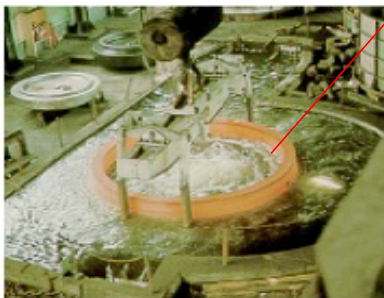
*Complex geometries*



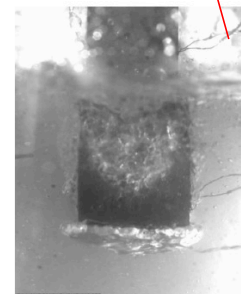
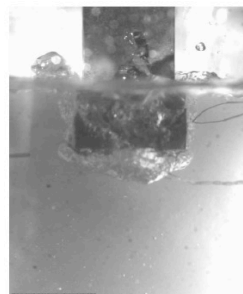
*Heat Treatment*

*Burner at ~50m/s*

*Quenching Process*



*Temperature > 1000° C*



*Turbulent mutliphase flows*



*Metal Casting*

# Challenges

« ...essentially driven by real industrial applications »

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## Involved physics

- Multiphase flows
- Turbulent boiling
- Phase change
- Liquid-gaz-solid flows
- Water “agitators”
- Surface tension
- High thermal gradients
- ...



## Process parameters:

- Orientation & position
- Size of the tank and the part
- Technology (jet, fall, ...)
- Stirring devices
- Fluid (water, oil, polymer ...)
- ...

## Other challenges

« ...essentially driven by real industrial applications »

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*Jet impinging for cooling*



*Aerodynamic performance of a buckled wing drone*



*Design of a new stratospheric airship*



*Analysis and understanding of complex fluid flows*

## Eulerian approach

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### Common points:

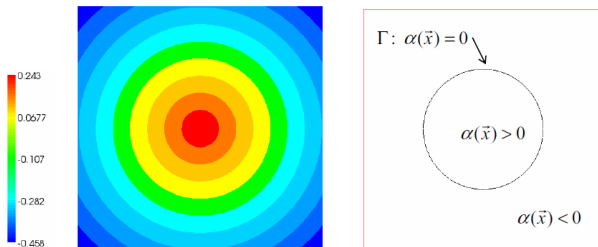
- Complex geometries
- Different phases (fluid-fluid, fluid-solid, ...)
- Need for optimization
- Accuracy at the interfaces
- Computational cost
- Repetitiveness
- ...

## Basic definition

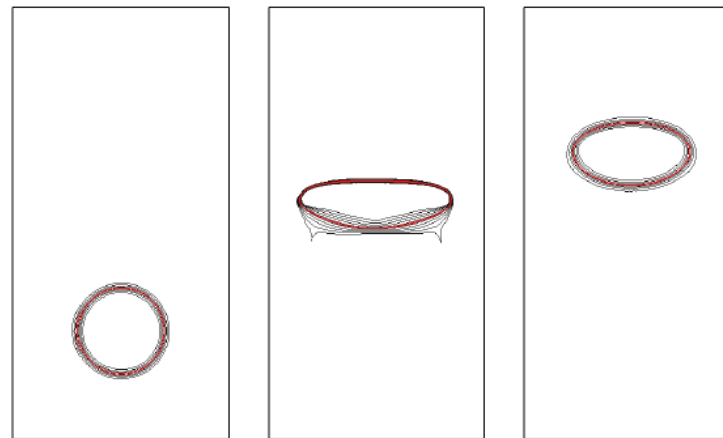
$$\alpha(X) = \begin{cases} -\text{dist}(X, \Gamma) & \text{if } X \in \Omega_1 \\ 0 & \text{if } X \in \Gamma \\ \text{dist}(X, \Gamma) & \text{if } X \in \Omega_2 \end{cases}$$

□ Transport equation  $\frac{\partial \alpha}{\partial t} + u \cdot \nabla \alpha = 0$

□ Hamilton-Jacobi problem  $\frac{\partial \alpha}{\partial \tau} + s(\alpha)(\|\nabla \alpha\| - 1) = 0$



Shape representation



Rising bubble

Without  
re-distancing

With  
re-distancing

- × Frequency for re-distancing
- × Mass conservation
- × Additional transport equation
- × Benefits from filtering
- × Benefits from stabilized FEM

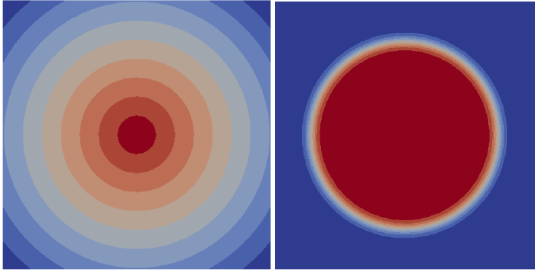
$$s(\alpha) = \frac{\alpha}{\sqrt{\alpha^2 + \epsilon^2}}$$



## Interface capturing

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- Apply a filter close to the interface, i.e. :  $\tilde{\alpha} = \frac{1}{1 + e^{-\frac{\alpha}{\varepsilon}}}$



Regular and filtered level set function

- Auto re-initialization equation

$$\left\{ \begin{array}{l} \frac{\partial \alpha}{\partial t} + u \cdot \nabla \alpha = 0 \\ \frac{\partial \alpha}{\partial \tau} + s(\alpha)(\|\nabla \alpha\| - 1) = 0 \end{array} \right.$$

- Shift the distance function restriction from  $\alpha$  to  $\tilde{\alpha}$

$$\|\nabla \alpha\| = 1 \quad \longrightarrow \quad \|\nabla \tilde{\alpha}\| = \frac{1}{\varepsilon}(1 - \alpha)\alpha$$

- Convective reactive level set method

$$\frac{\partial \tilde{\alpha}}{\partial t} + (u + \lambda U) \cdot \nabla \tilde{\alpha} = s(\tilde{\alpha}) \frac{\lambda}{\varepsilon} (1 - \tilde{\alpha}) \tilde{\alpha}$$

- Time discretisation

$$\frac{3\tilde{\alpha}^{n+1} - 4\tilde{\alpha}^n + \tilde{\alpha}^{n-1}}{2\Delta t} + (u^{n+1} + \lambda U^n) \cdot \nabla \tilde{\alpha}^{n+1} - s(\tilde{\alpha}) \frac{\lambda}{\varepsilon} (1 - \tilde{\alpha}^n) \tilde{\alpha}^{n+1} = 0$$

one choice among many for linearization

- Stabilized finite element variational formulation

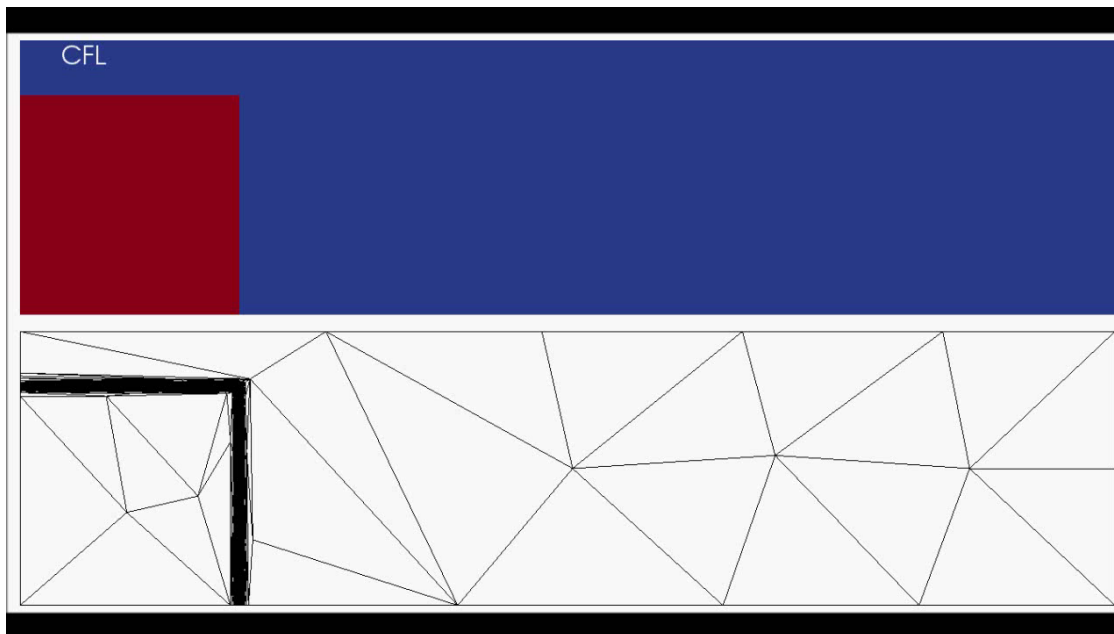
$$\left( \frac{3\tilde{\alpha}_h^{n+1} - 4\tilde{\alpha}_h^n + \tilde{\alpha}_h^{n-1}}{2\Delta t}, \omega_h \right)_{\Omega} + ([u_h^{n+1} + \lambda U_h^n] \cdot \nabla \tilde{\alpha}_h^{n+1}, \omega_h)_{\Omega} - \left( s(\tilde{\alpha}_h) \frac{\lambda}{\varepsilon} (1 - \tilde{\alpha}_h^n) \tilde{\alpha}_h^{n+1}, \omega_h \right)_{\Omega} \\ + \sum_K (\mathcal{R}(\tilde{\alpha}_h^{n+1}), \tau^n [u_h^{n+1} + \lambda U_h^n] \cdot \nabla \omega_h)_K + \sum_K (\mathcal{R}(\tilde{\alpha}_h^{n+1}), \tau^n | s(\tilde{\alpha}_h) \frac{\lambda}{\varepsilon} (1 - \tilde{\alpha}_h^n) \tilde{\alpha}_h^{n+1} | \cdot \omega_h)_K = 0,$$

$$\mathcal{R} = \frac{\partial \tilde{\alpha}_h}{\partial t} - (u_h + \lambda U_h \nabla \tilde{\alpha}_h) - s(\tilde{\alpha}_h) \frac{\lambda}{\varepsilon} (1 - \tilde{\alpha}_h) \tilde{\alpha}_h$$

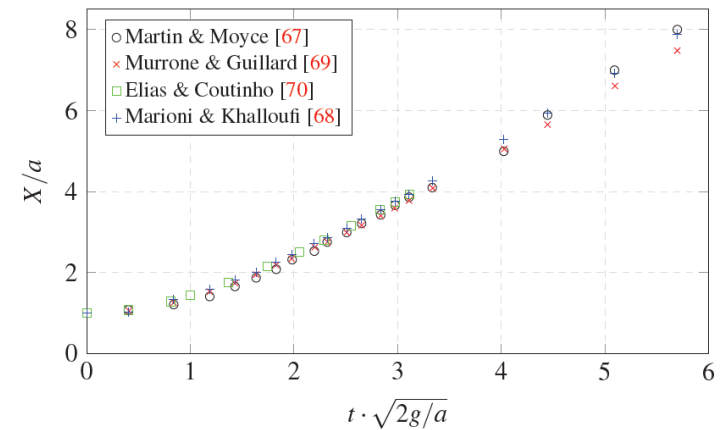
## Illustration: collapse of a water column

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Two-fluid flow: water-air



Validation using 5000 nodes



- Navier-Stokes ?
- Anisotropic meshing?
- Other physics (fluid-solid, complex fluids)?

M Khalloufi, Y Mesri, R Valette, E Massoni, E Hachem, High fidelity anisotropic adaptive variational multiscale method for multiphase flows with surface tension, *Computer Methods in Applied Mechanics and Engineering*, Vol 307, 44-67, 2016

L. Marioni, M. Khalloufi, F. Bay, E. Hachem, Two-fluid flow under the constraint of external magnetic field: revisiting the dam-break benchmark, *International Journal of Numerical Methods for Heat & Fluid Flow*, Vol. 27, pp. 2565-2581, 2017

# The Navier-Stokes equations

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VMS: Variational MultiScale

- VMS methods consider large scales which are defined by projection into appropriate spaces
- Models both velocity and pressure unresolved scales
- Similarity with the implicit version of LES

Variational Multiscale formulation:  $\mathbf{v} = \mathbf{v}_h + \mathbf{v}' \quad p = p_h + p'$

- the use of equal order continuous interpolations
- preventing from oscillations due to convection dominated flows

[T.J.R. Hugues et al., 1998]  
 [L.P. Franca, A. Nesliturk, 2001]  
 [R. Codina, 2002]  
 [V. Gravemeier, W.A. Wall, E. Ramm, 2004]  
 [A. Masud, R.A. Khurram, 2006]  
 [E. Hachem et al., 2010]  
 ...

find  $(\mathbf{v}_h + \mathbf{v}', p_h + p') \in V_h \oplus V' \times P_h \oplus P'$  such that

$$\begin{aligned} (\rho \delta_t(\mathbf{v}_h + \mathbf{v}'), \mathbf{w}_h + \mathbf{w}') + (\rho(\mathbf{v}_h + \mathbf{v}') \cdot \nabla(\mathbf{v}_h + \mathbf{v}'), \mathbf{w}_h + \mathbf{w}') - (p_h + p', \nabla \cdot (\mathbf{w}_h + \mathbf{w}')) \\ + 2(\eta \varepsilon(\mathbf{v}_h + \mathbf{v}'), \varepsilon(\mathbf{w}_h + \mathbf{w}')) = \langle \mathbf{f}, \mathbf{w}_h + \mathbf{w}' \rangle \\ (q_h + q', \nabla \cdot (\mathbf{v}_h + \mathbf{v}')) = 0 \end{aligned}$$

for all  $(\mathbf{w}_h + \mathbf{w}', q_h + q') \in V_{h,0} \oplus V'_0 \times P_{h,0} \oplus P'_0$ .

The subscales are approximated within each element K by:

$$\mathbf{v}' = \alpha_v \Pi'_v(\mathcal{R}_v), \quad p' = \alpha_p \Pi'_p(\mathcal{R}_p),$$

# The Navier-Stokes equations

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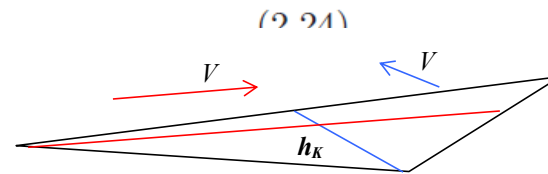
Variational MultiScale method:

- approximate the fine scale within each element  $K$
- inserting the expressions of the subscales in the coarse scale equations
- fully implicit resolution

$$\begin{aligned}
 & \frac{(\rho \delta_t \mathbf{v}_h, \mathbf{w}_h) + (\rho \mathbf{v}_h \cdot \nabla \mathbf{v}_h, \mathbf{w}_h) - (p_h, \nabla \cdot \mathbf{w}_h) + 2(\eta \varepsilon(\mathbf{v}_h), \varepsilon(\mathbf{w}_h))}{\sum_K \alpha_v (\rho \delta_t \mathbf{v}_h + \rho \mathbf{v}_h \cdot \nabla \mathbf{v}_h + \nabla p_h - \nabla \cdot (2\eta \varepsilon(\mathbf{v}_h)), \rho \mathbf{v}_h \cdot \nabla \mathbf{w}_h + \nabla \cdot (2\eta \varepsilon(\mathbf{w}_h)))_K} \\
 & + \sum_K \alpha_p (\nabla \cdot \mathbf{v}_h, \nabla \cdot \mathbf{w}_h) \\
 & = \langle \mathbf{f}, \mathbf{w}_h \rangle + \sum_K \alpha_v (\mathbf{f}, \rho \mathbf{v}_h \cdot \nabla \mathbf{w}_h + 2\eta \nabla \cdot \varepsilon(\mathbf{w}_h))_K \\
 & \frac{(q_h, \nabla \cdot \mathbf{v}_h) + \sum_K \alpha_v (\rho \delta_t \mathbf{v}_h + \rho \mathbf{v}_h \cdot \nabla \mathbf{v}_h + \nabla p_h - \nabla \cdot (2\eta \varepsilon(\mathbf{v}_h)), \nabla q_h)_K}{\sum_K \alpha_v (\mathbf{f}, \nabla q_h)_K}
 \end{aligned}$$

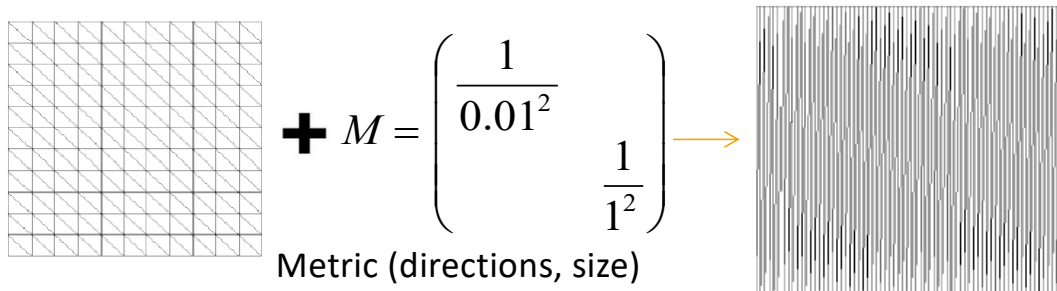
and the stabilization parameters:

$$\alpha_v = \left[ \left( \frac{c_1 \eta}{\rho h^2} \right)^2 + \left( \frac{c_2 \|\mathbf{v}_h\|_K}{h} \right)^2 \right]^{-1/2} \quad \alpha_p = \left[ \left( \frac{\eta}{\rho} \right)^2 + \left( \frac{c_2 \|\mathbf{v}_h\|_K h}{c_1} \right)^2 \right]^{1/2} \quad h_K = \frac{2|\mathbf{v}_h|}{\sum_{i=1}^{N_K} |\mathbf{v}_h \cdot \nabla \varphi_i|}$$



[E. Hachem et. al, JCP 2010]  
 [E. Hachem et. al, CMAME, 2016]  
 [P. Meliga & E. Hachem, JCP, 2018]

## □ Metric-based anisotropic mesh adaptation



### □ Motivation 1

- Enables to capture scale heterogeneities
- Enables to deal with discontinuities or gradients
- Crucial for boundary layers, shock waves, ...
- Crucial for complex geometry: curvature, sharp angles,...



### □ Motivation 2

- Dynamic Mesh adaptation
- A posteriori error estimator
- Multi criteria adaptation
- Control (i.e. number of elements)
- Error analysis

P. J. Frey and F. Alauzet. Anisotropic mesh adaptation for cfd computations. Computer Methods in Applied Mechanics and Engineering, 194(48-49):5068–5082, 2005.

J.-F. Remacle, X. Li, M.S. Shephard, and J.E. Flaherty. Anisotropic adaptive simulation of transient flows. International Journal for Numerical Methods in Engineering, 62:899–923, 2005

H. Beaugendre R. Abgrall and C. Dobrzynska. An immersed boundary method using unstructured anisotropic mesh adaptation combined with level-sets and penalization techniques. Journal of Computational Physics, 257:83–101, 2014



## Combining mesh adaptation with SFEM

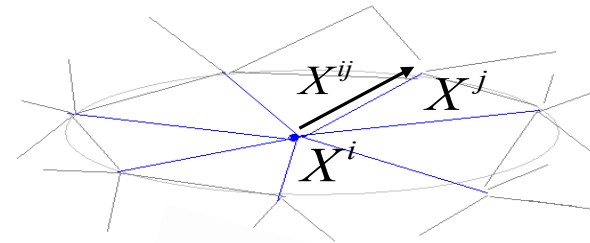
15

- Metric construction

$$\widetilde{\mathbb{M}}^i = \frac{|\Gamma(i)|}{d} \left( \widetilde{\mathbb{X}}^i \right)^{-1}$$

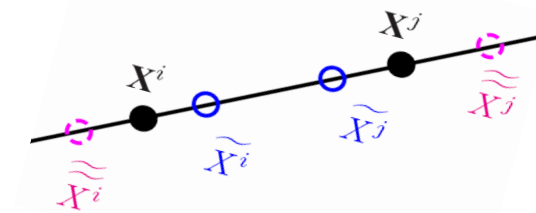
- Candidate for directions:

$$\mathbb{X}^i = \frac{d}{|\Gamma(i)|} \sum_{j \in \Gamma(i)} \mathbf{X}^{ij} \otimes \mathbf{X}^{ij}$$



- Control the size of each edge  $X^{ij}$  using a stretching factor:

$$\widetilde{\mathbf{X}}^{ij} = s_{ij} \mathbf{X}^{ij}$$



- Drive these stretching factors using an edge based error estimator:

$$s_{ij} = \left( \frac{e}{e(N)} \right)^{-\frac{1}{2}} = \left( \frac{\sum_i n^i(1)}{N} \right)^{\frac{2}{d}} e_{ij}^{-1/2}$$

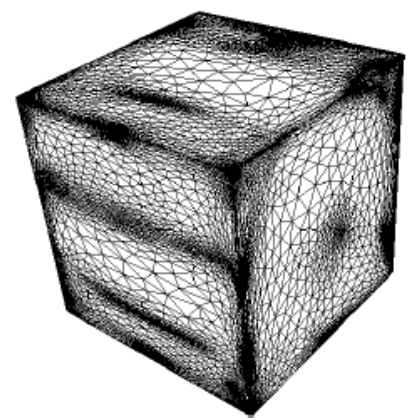
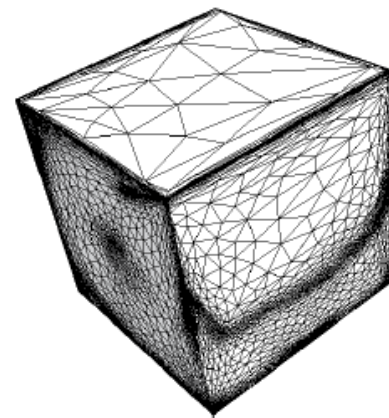
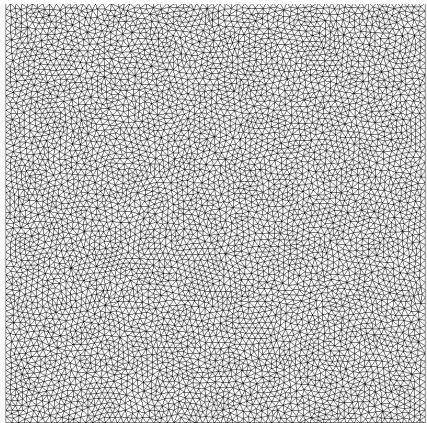
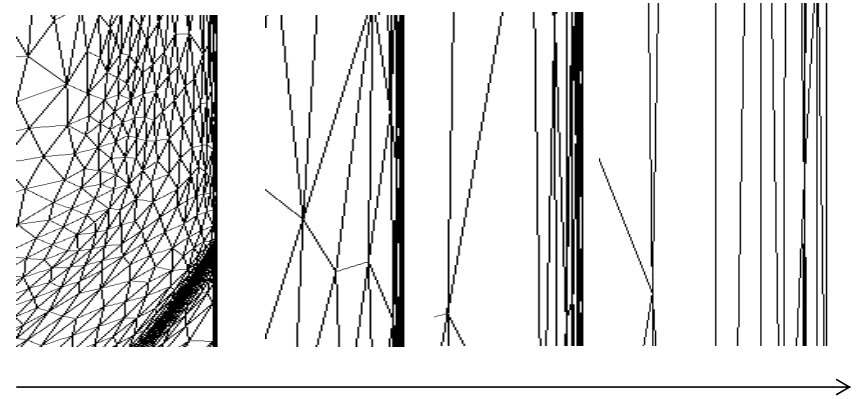
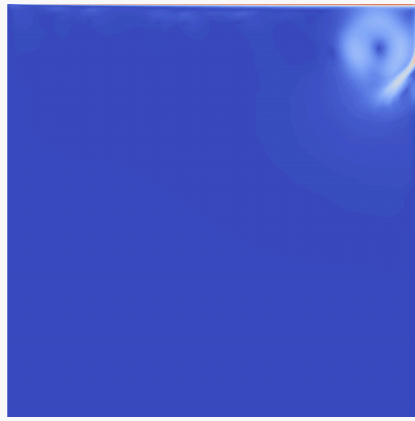
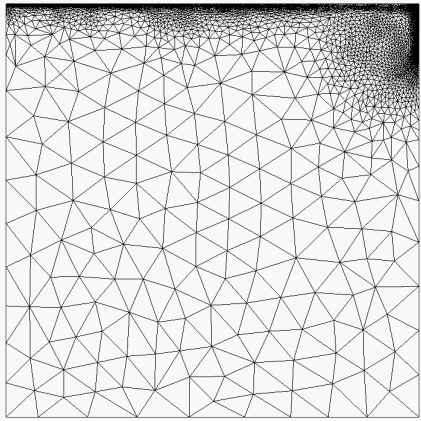
- Multi criteria mesh adaptation

$$\mathbf{v}(\mathbf{x}^i) = \left\{ \frac{\hat{\alpha}}{E}, \frac{v^i}{\|v^i\|}, \frac{\|v^i\|}{\max_j \|v^j\|}, \frac{\eta_{eff}}{\max(\eta_{eff})} \right\}$$

T. Coupez, E. Hachem, Solution of high-Reynolds incompressible flow with stabilized finite element and adaptive anisotropic meshing, Computer methods in applied mechanics and engineering, Vol. 267, 65-85, 2013

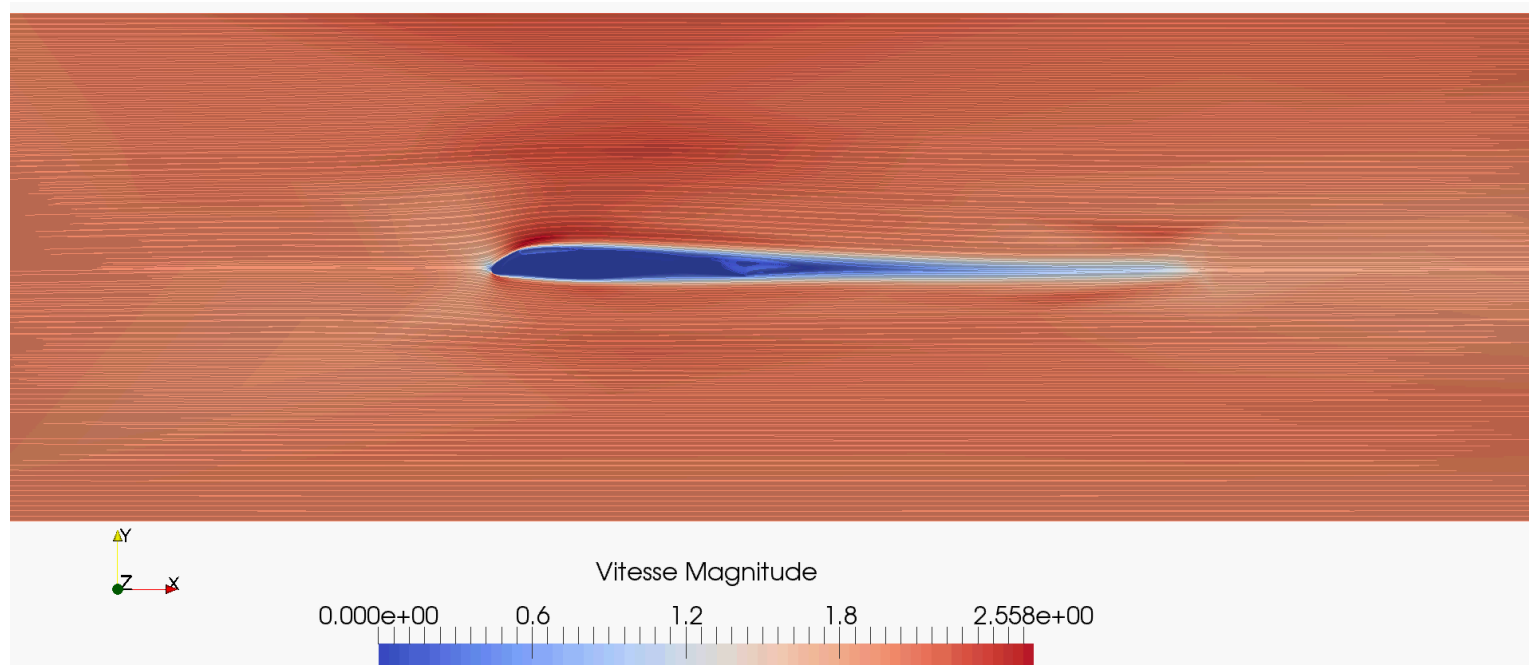
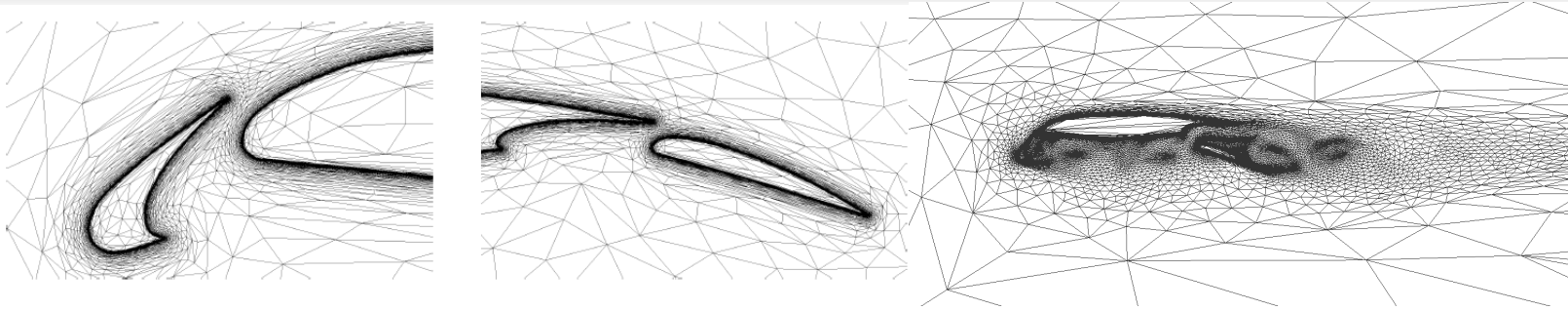
## Revisiting the lid driven cavity

16

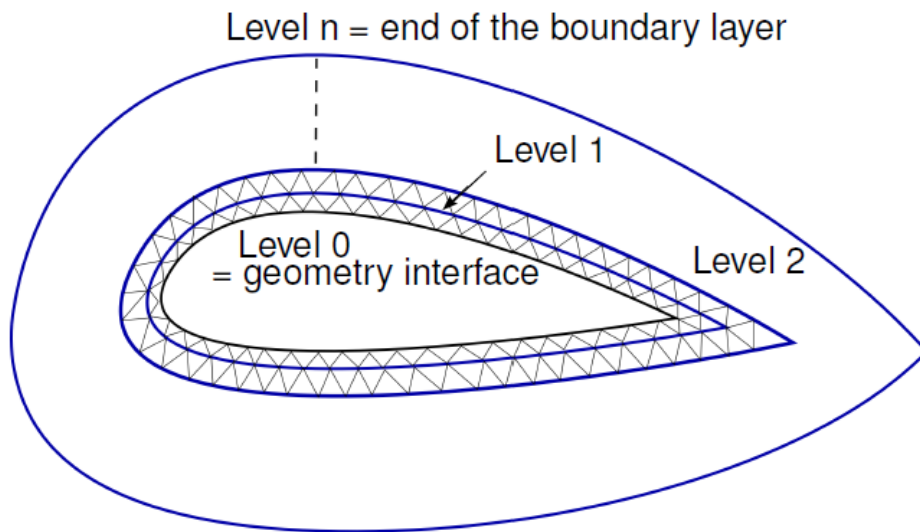


## Combining **BLM** (boundary layer metric) and **EBM** (edge-based metric)

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### Multi-levelset framework

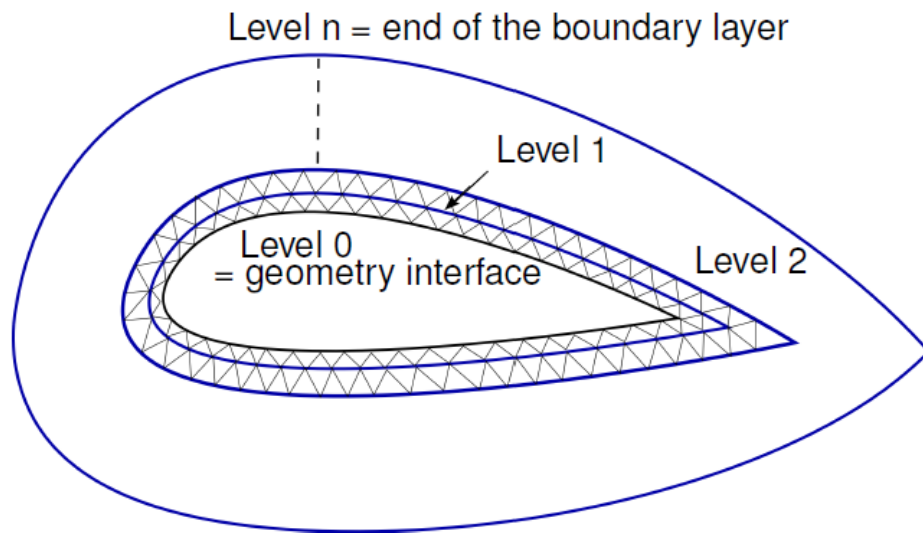


$$\begin{aligned}\text{Levels}[k] &= \text{Levels}[k-1] + h_{\min} \times \alpha^{k-1} \\ &= \text{Levels}[0] + h_{\min}(1 - \alpha^k)/(1 - \alpha)\end{aligned}$$

$$\mathbf{M} = \frac{1}{h_n^2} \mathbf{N} \times \mathbf{N}^T + \frac{1}{h_t^2} (\mathbf{I}_d - \mathbf{N} \times \mathbf{N}^T)$$

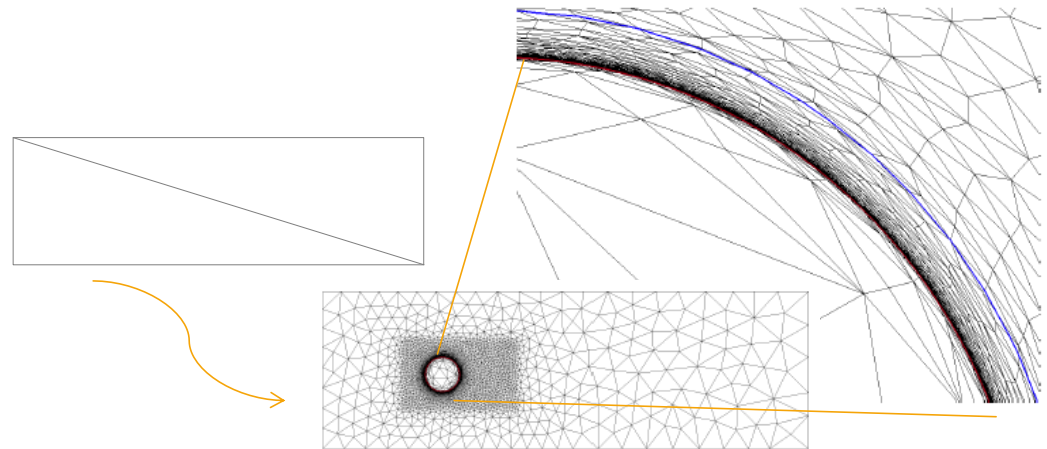
- ❑ Tangential mesh size must depend on the geometry and consequently:  
→ shape, curvature and the complexity of the geometry
- ❑ In three dimensions, the geometry can have different behavior in its two tangential directions.
- ✓ We refer to the geometry curvature to define properly the tangential directions and associated mesh sizes
- ✓ It will allow us to define properly the anisotropic ratio and to ensure that the interface is smoothly and well described.

### Multi-levelset framework



$$\begin{aligned}\text{Levels}[k] &= \text{Levels}[k-1] + h_{\min} \times \alpha^{k-1} \\ &= \text{Levels}[0] + h_{\min}(1 - \alpha^k)/(1 - \alpha)\end{aligned}$$

- Boundary layer at the geometry interface
- Ensure a smooth mesh size transition
- Build a quasi-structured mesh at the interface

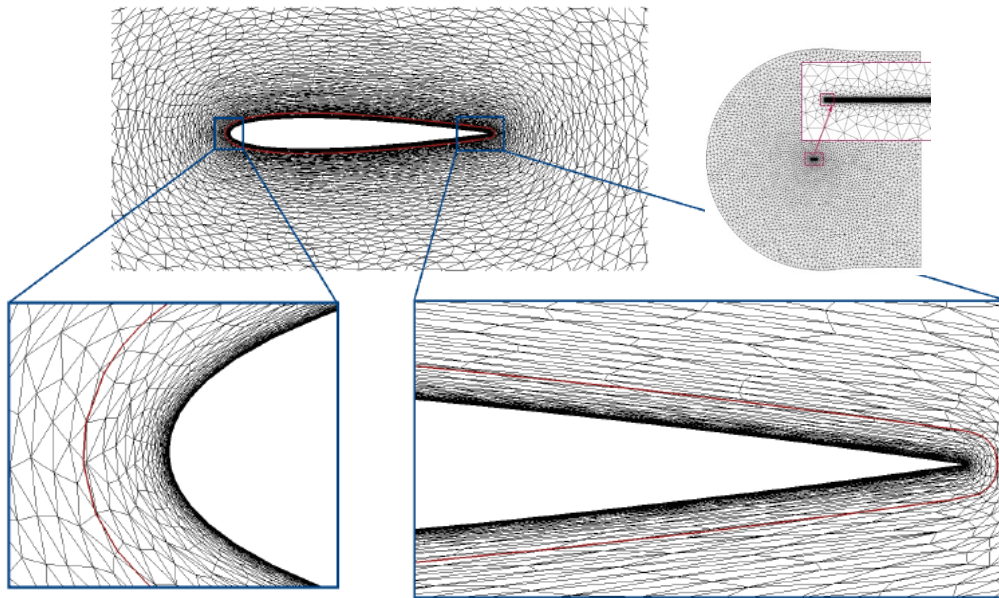




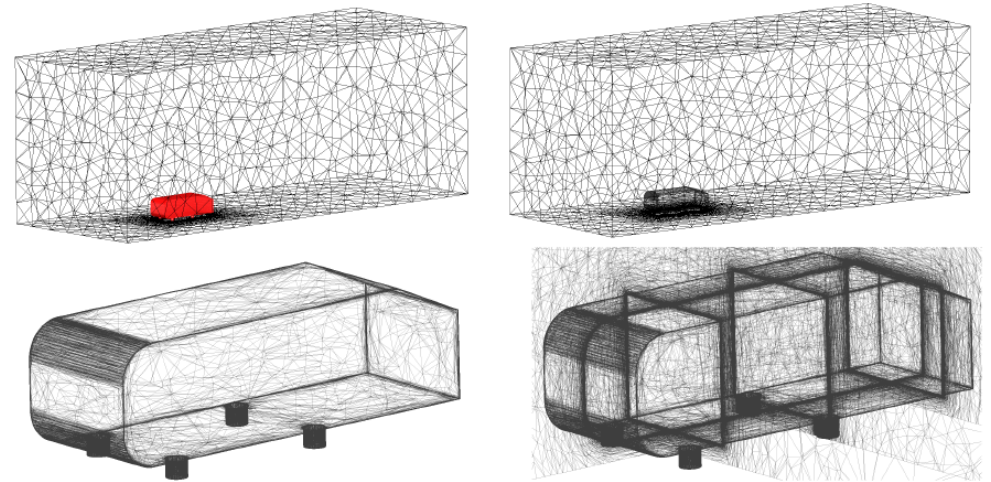
## BLM (boundary layer metric)

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NACA0012:  $Re = 6 \cdot 10^6$



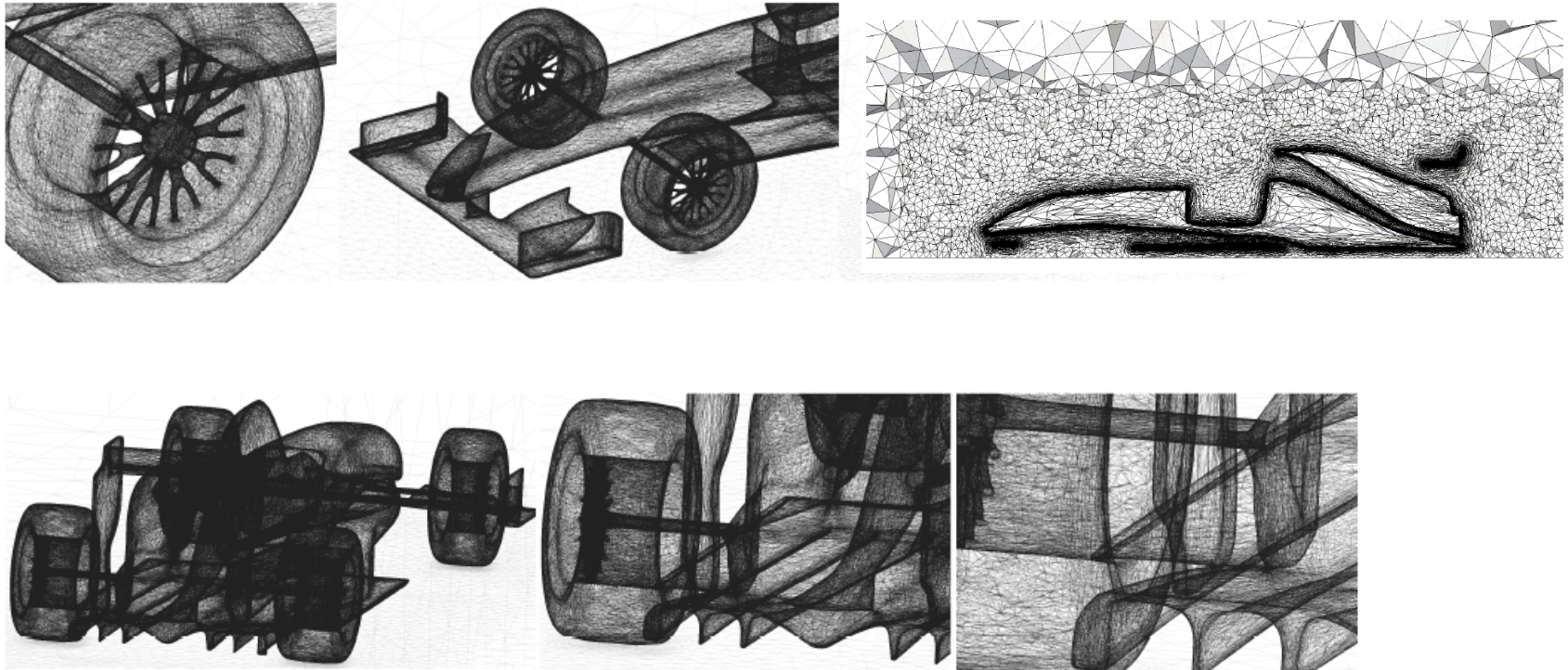
$y_0^+ = 1, h_{\min} = 4.46 \cdot 10^{-6}$



2.167.057 elements - 378.259 nodes - 20 cores  
 $Re = 4.25 \cdot 10^6 - h_{\min} = 3 \cdot 10^{-4}$

## BLM (boundary layer metric)

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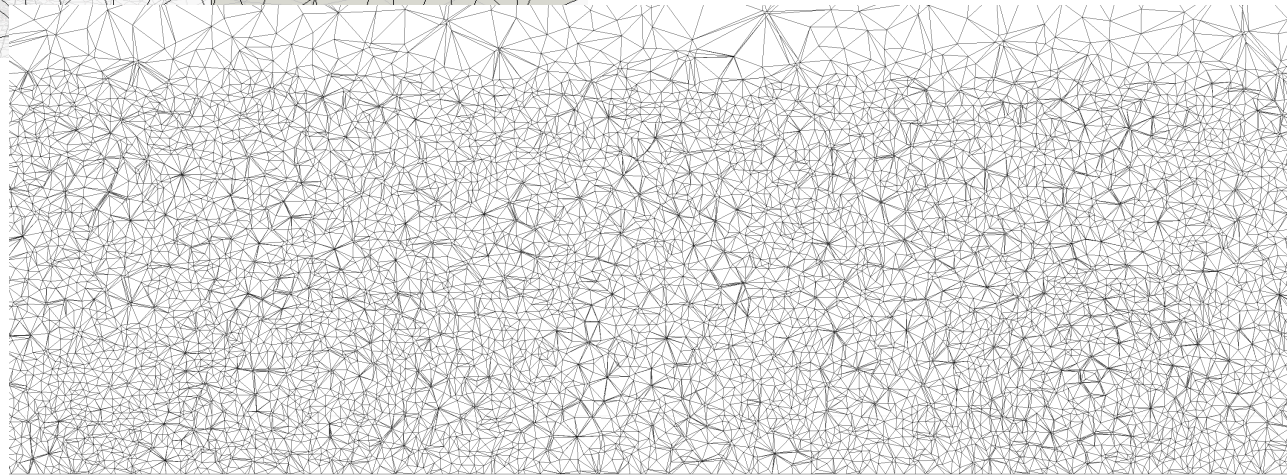
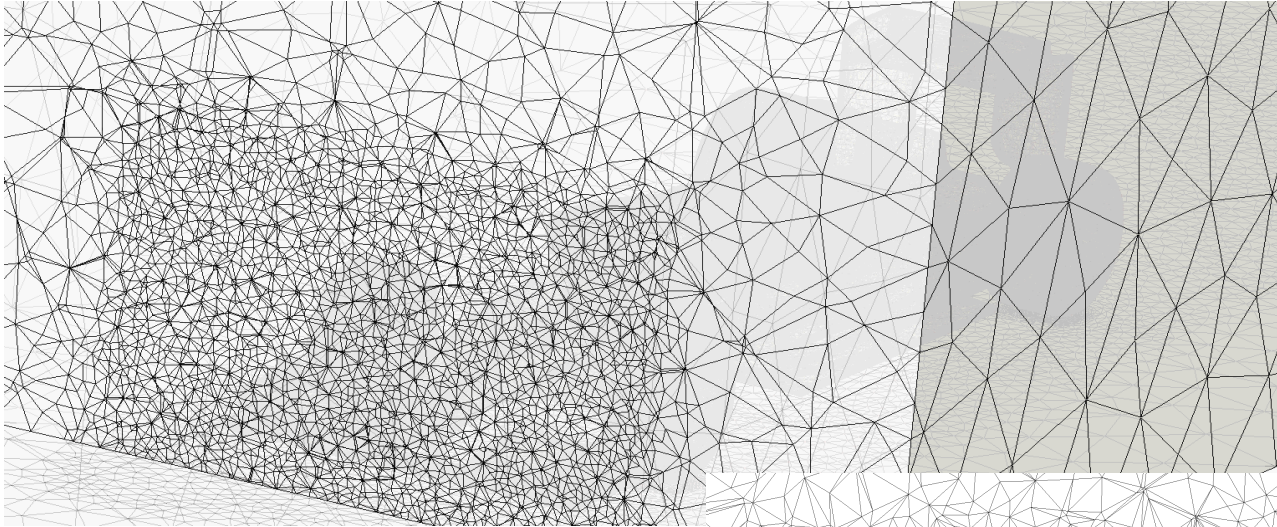
21.026.520 elements - 3.602.483 nodes - 40 cores

$$\text{Re} = 2 \cdot 10^7 - h_{\min} = 6 \cdot 10^{-4}$$



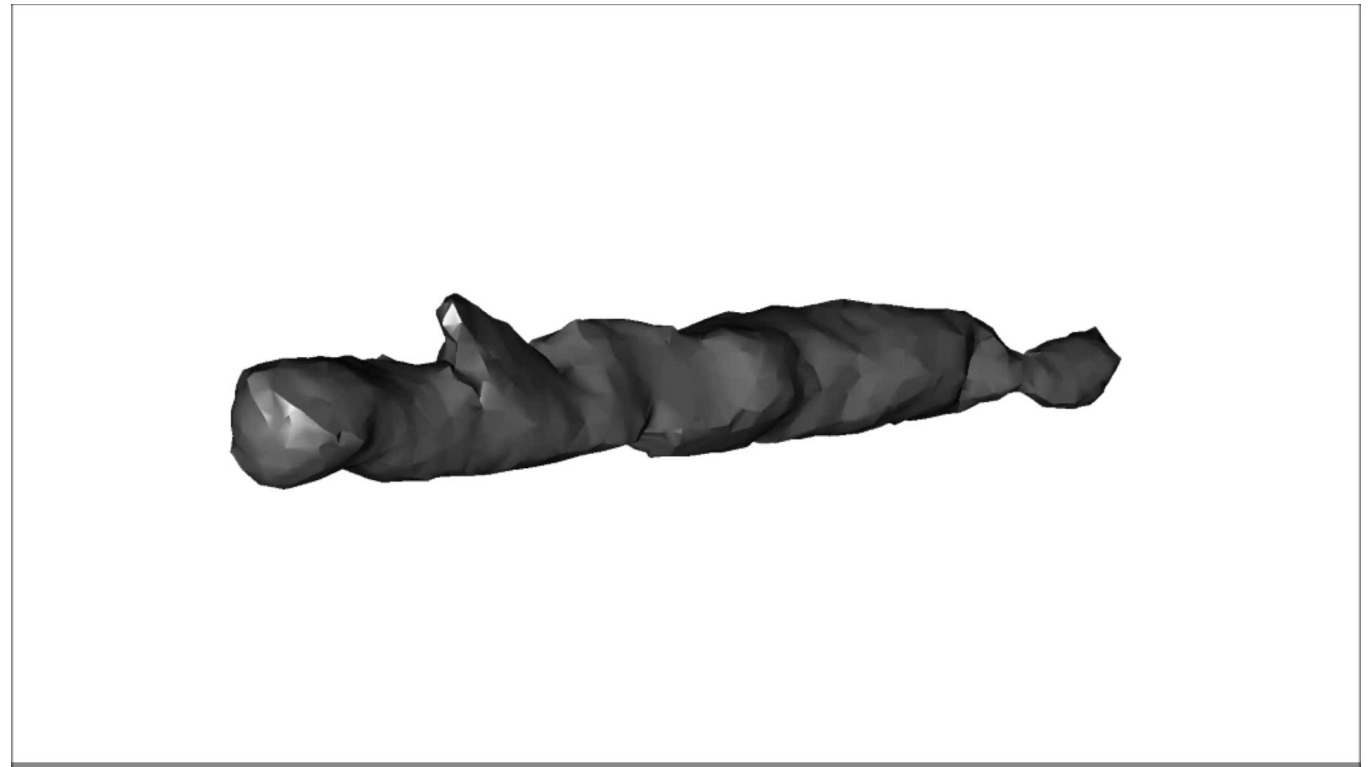
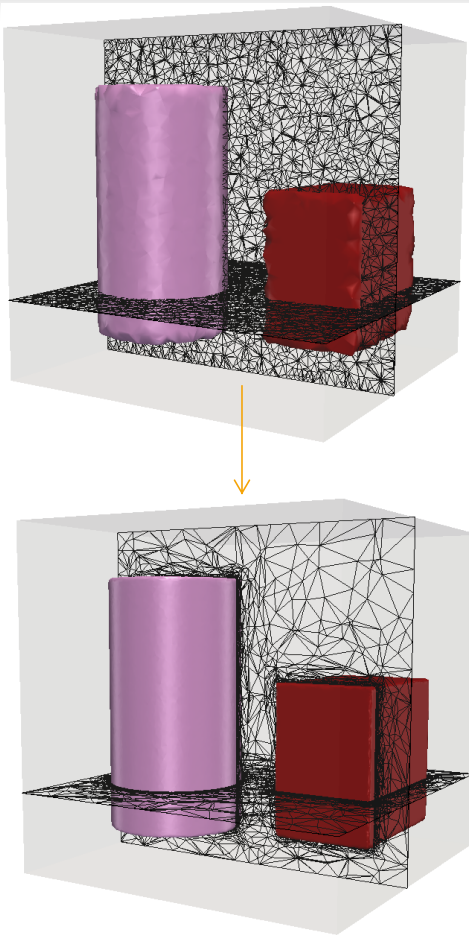
## BLM (boundary layer metric)

22



## An iterative process – immersed of complex geometries

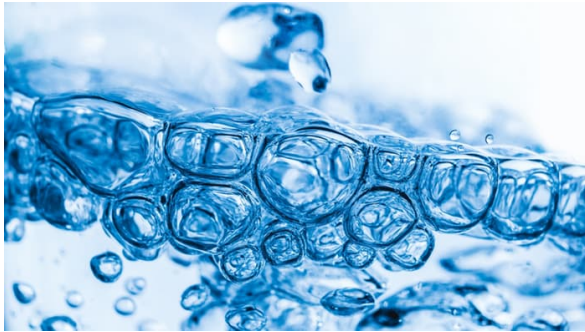
23



*Isovalues of the distance function on an anisotropic mesh*

## Towards a general Eulerian two-fluid framework

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Liquid-vapor flows



Yield stress flows



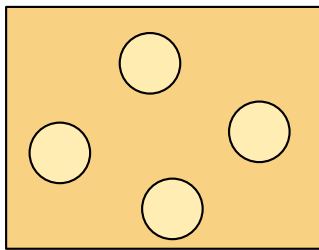
Granular flows



# Eulerian two-fluid framework

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Continuum approach for granular flow

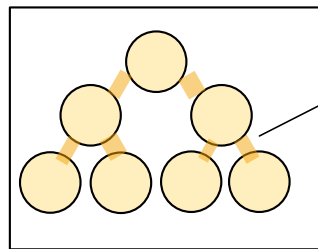


liquid

→ Newtonian behaviour

Newtonian

$$\tau = 2\eta D(u)$$



Capillary bridges

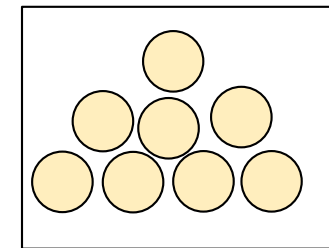
Intermediate state

→ Yield stress fluid

Bingham

$$\tau = \left( 2\eta + \frac{\tau_0}{\dot{\gamma}} \right) D(u)$$

$$\dot{\gamma} = \|D(u)\|$$



Granular material

→ Pressure-dependent behaviour

Granular

$$\tau = 2 \left( \frac{\mu - \mu_s}{I + I_0} I + \frac{\mu_s}{\dot{\gamma}} \right) p D(u)$$

Jop et al., Nature, 2006

Numerical issues

$$p \rightarrow 0$$

$$\dot{\gamma} \rightarrow 0$$

surface tension

high density / viscosity ratios

Surface tension

$$\Delta t < (\Delta x)^{\frac{3}{2}} \sqrt{\frac{\bar{\rho}}{2\pi\gamma}}$$

$$f_{ST} = -\gamma\kappa\delta(\alpha)\mathbf{n} - \gamma\delta(\alpha)\Delta t \left( \frac{\partial^2 \mathbf{u}}{\partial \mathbf{n}^2} + \kappa \frac{\partial \mathbf{u}}{\partial \mathbf{n}} - \nabla^2 \mathbf{u}^{n+1} \right)$$

M. Khalloufi, Y. Mesri, R. Valette, E. Hachem, CMAME 2016

Papanastasiou regularization method

[T. Papanastasiou 1987]

$$\eta_{eff} = \eta_p + \frac{\tau_0}{\dot{\varepsilon}} [1 - \exp(-m\dot{\varepsilon})]$$

m : Papanastasiou regularization coefficient

S. Riber, Y. Mesri, R. Valette, E. Hachem, Computers & Fluids, 2016

Extended to 3D free surface flows

[M. Bercovier, 1980]

$$\eta_{eff} = \min \left( \eta_{air}, \eta_f(p, \|\dot{\gamma}\|) + \frac{\tau_0(p)}{\sqrt{\|\dot{\gamma}\|^2 + \|\dot{\gamma}\|_{min}^2}} \right)$$

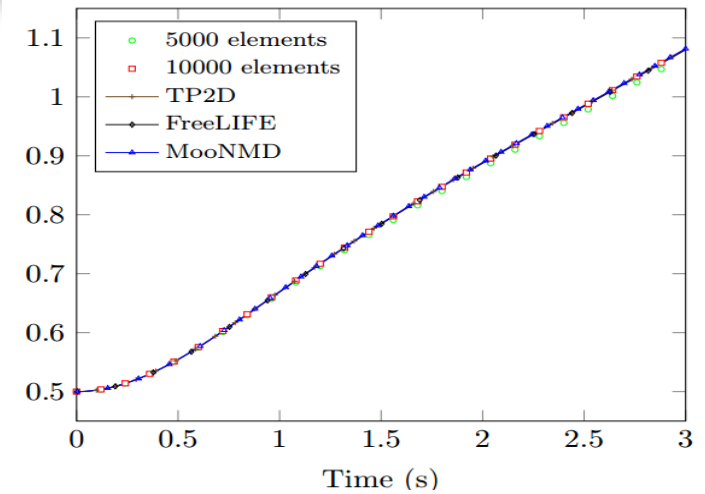
R. Valette, S. Riber, A.S. Pereira, M. Khalloufi, L. Sardo, E. Hachem, J. Comp. Phys., submit 2018

## Numerical test 1: rising bubble

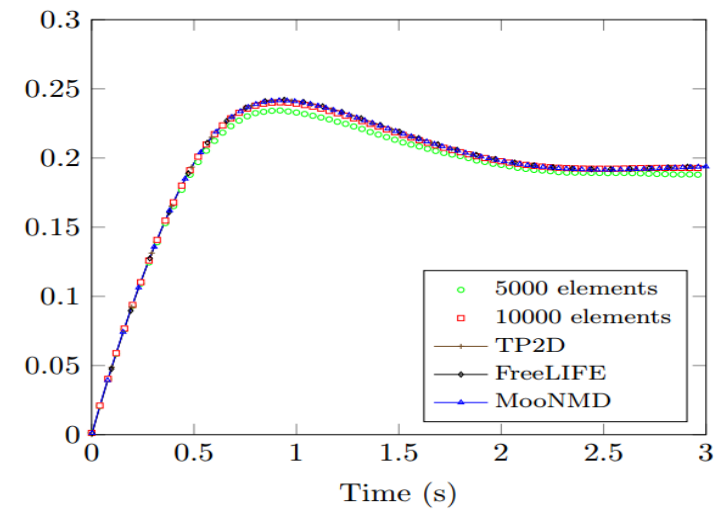
27



Position of the  
center of mass

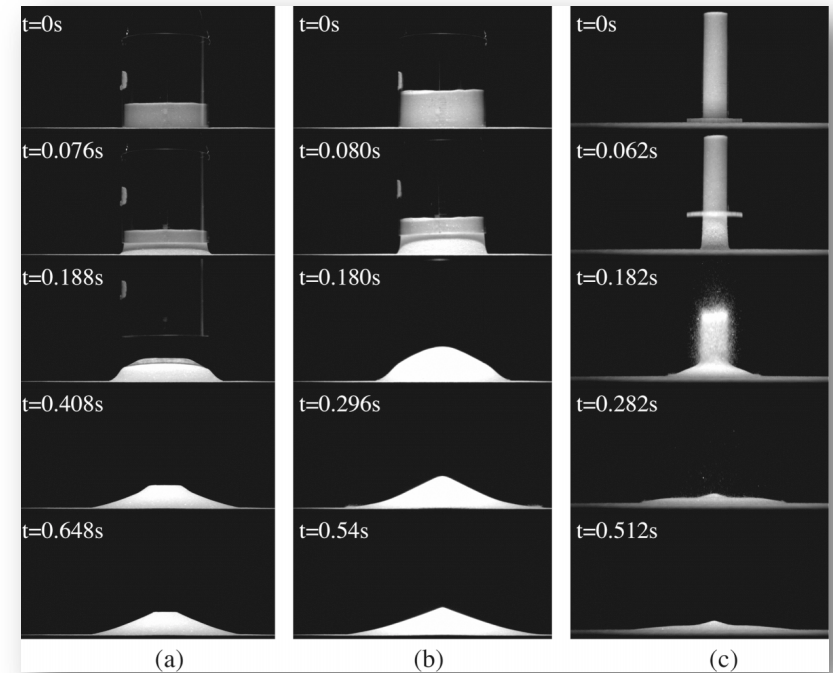
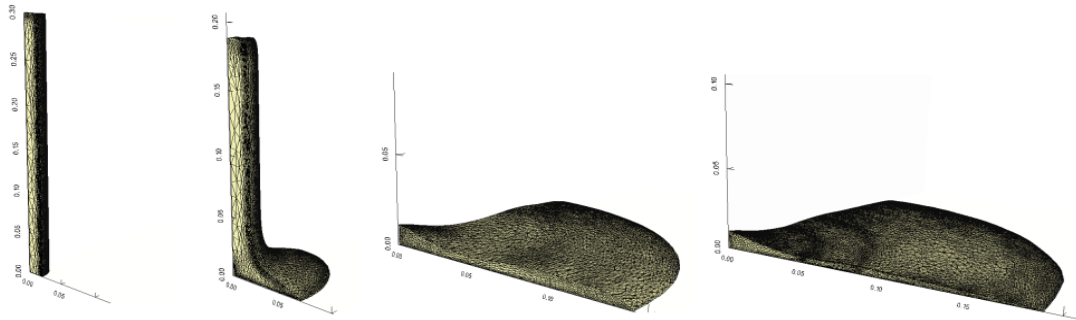
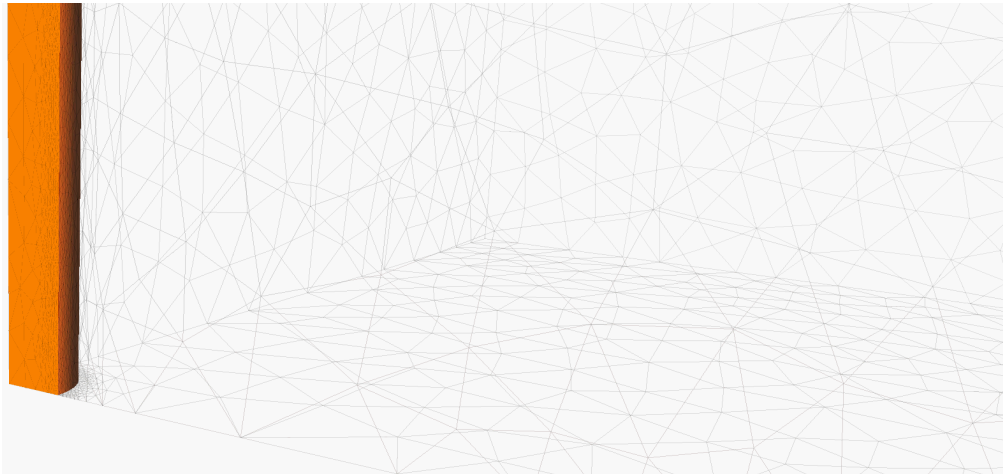


Rising velocity



## Numerical test 2: collapse of a granular column

- 3D dam break

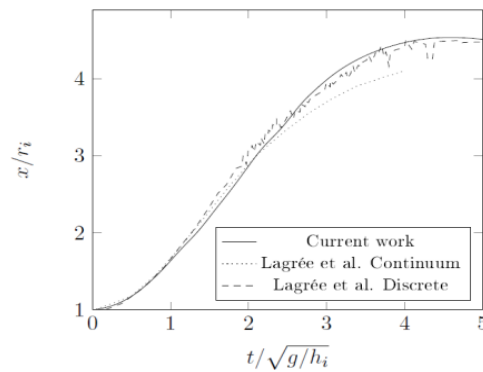


R. Valette, S Riber, L Sardo, R Castellani, F Costes, N Vriend, E Hachem, Sensitivity to the rheology and geometry of granular collapses by using the  $\mu(I)$  rheology, Computers & Fluids 191, 104260

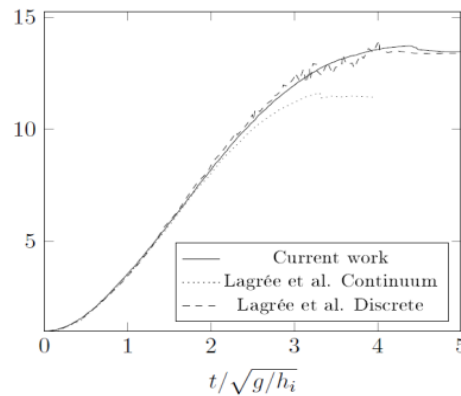
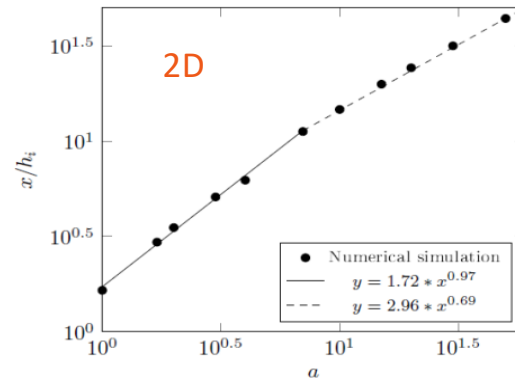
## Numerical test 2: collapse of a granular column

29

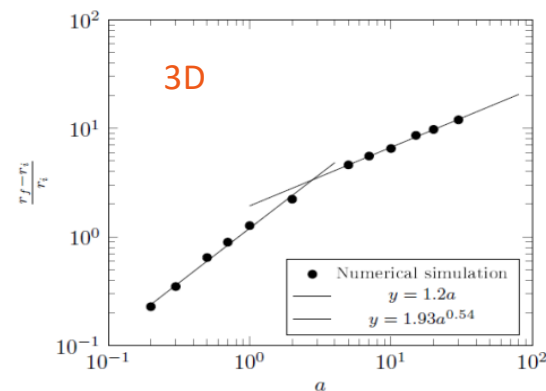
### Comparisons with the discrete method



(b)  $a = 1.42$

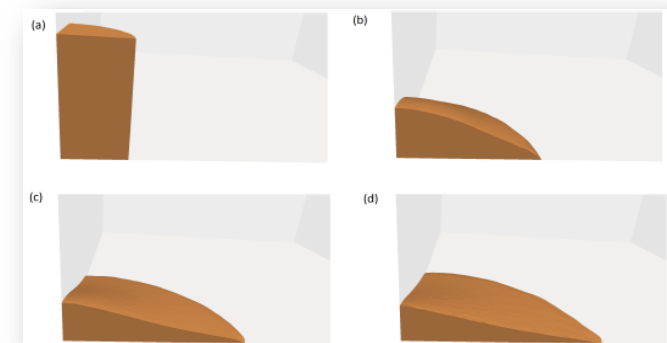


(c)  $a = 6.26$

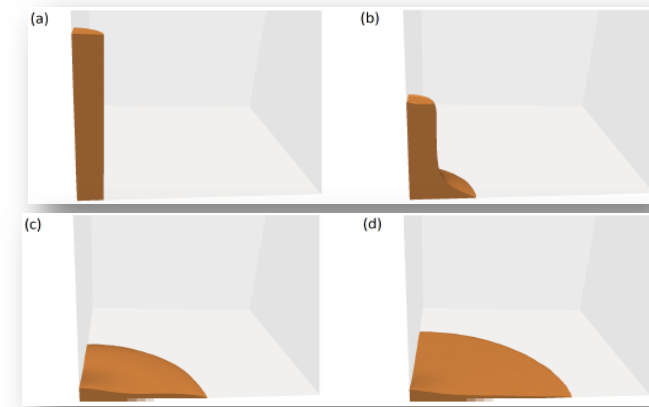


Lagrée et al., J. Fluid Mech. 2011

Aspect ratio  $a=2$



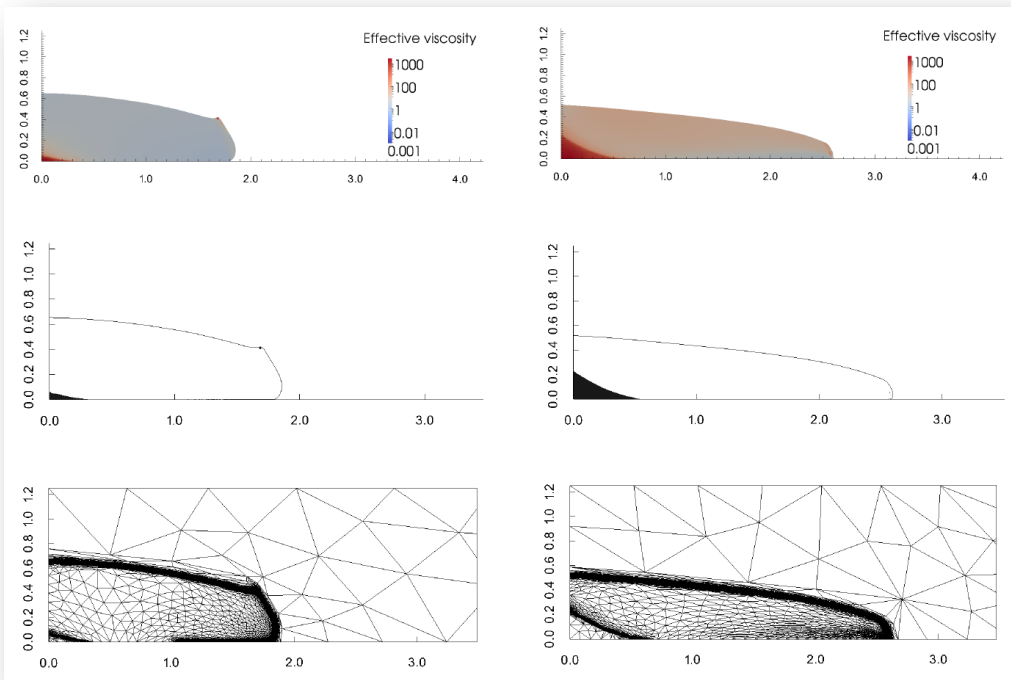
Aspect ratio  $a=7$



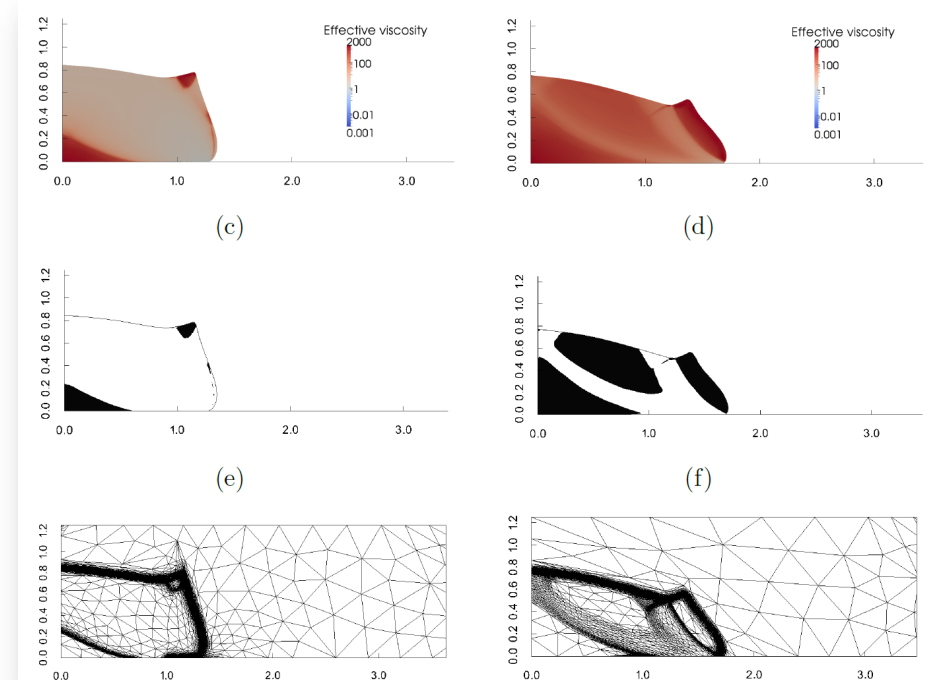
## Numerical test 3: Collapse of a Bingham column

30

- 2D and 3D Bingham dam break problem: effective viscosity, yielded regions and adapted mesh



Aspect ratio  $a = 1$   
 $Bn = 0.03$  at  
 $t = 10$  and  $100$



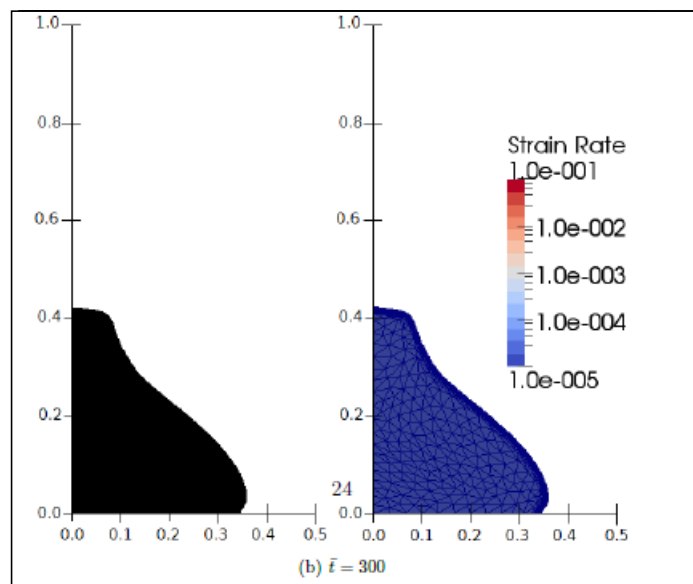
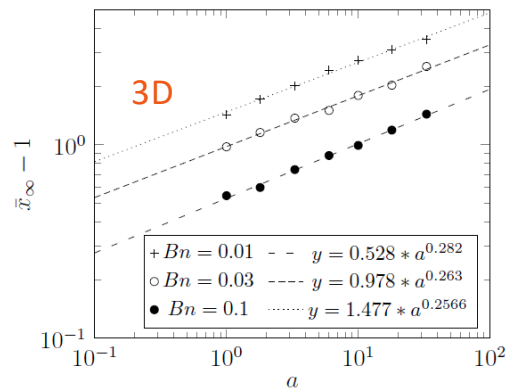
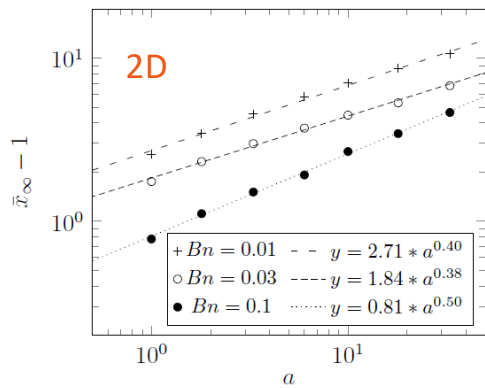
Aspect ratio  $a = 1$   
 $Bn = 0.1$   
 $t = 10$  and  $100$



## Numerical test 3: Collapse of a Bingham column (new benchmark)

31

- 2D and 3D Bingham dam break problem: effective viscosity, yielded regions and adapted mesh



2D and 3D Bingham collapse ( $a=10$ ) with  $Bn = 0:1$  at  $\bar{t} = 1; 100; 200; 300$

## Numerical test 4: Compressed material (new benchmark)

32

- Different buckling regime



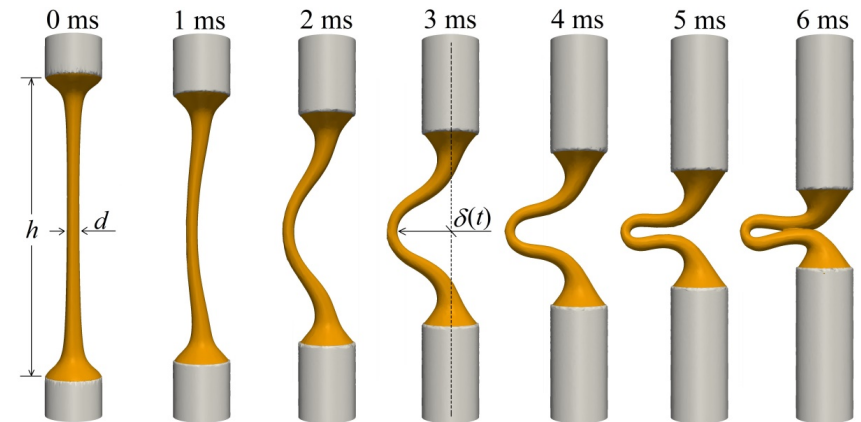
(a)  $Re=0.0013$ ;  $Ca=200$ ;  $d/h=0.04$



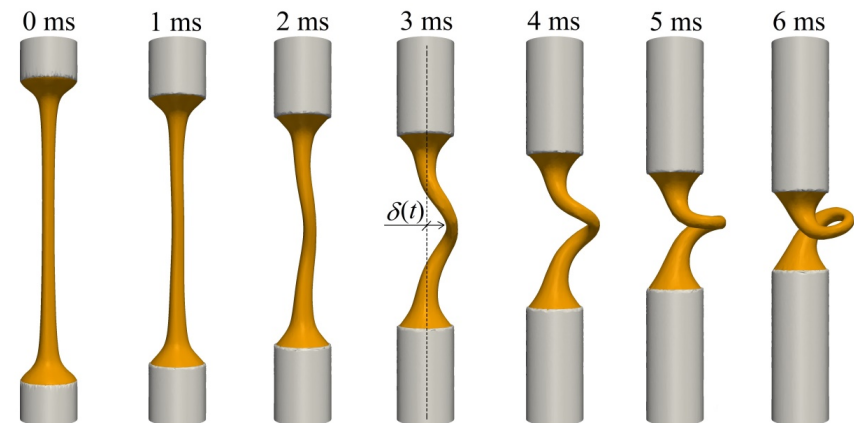
(b)  $Re=0.13$ ;  $Ca=200$ ;  $d/h=0.04$



(a)  $Re = 0.0013$  ;  $Ca = 200$  ;  $d_0/h = 0.04$



(b)  $Re = 0.13$  ;  $Ca = 200$  ;  $d_0/h = 0.04$

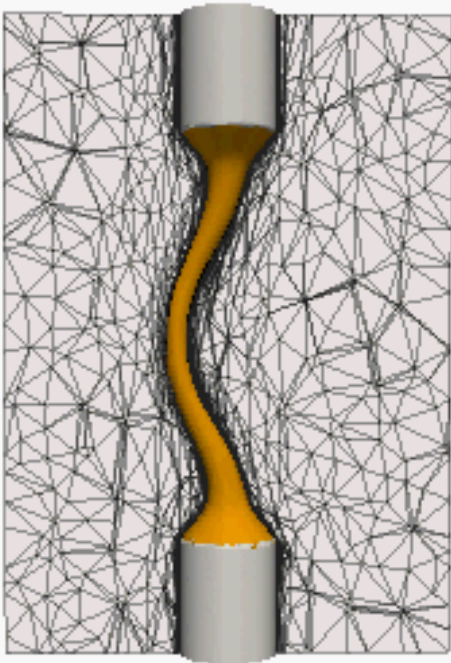


## Numerical test 4: Compressed material (new benchmark)

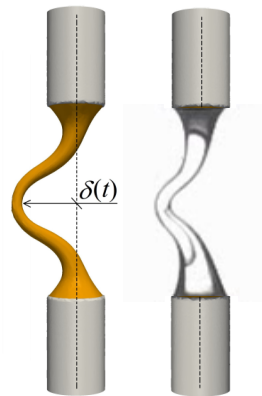
33

- Main results

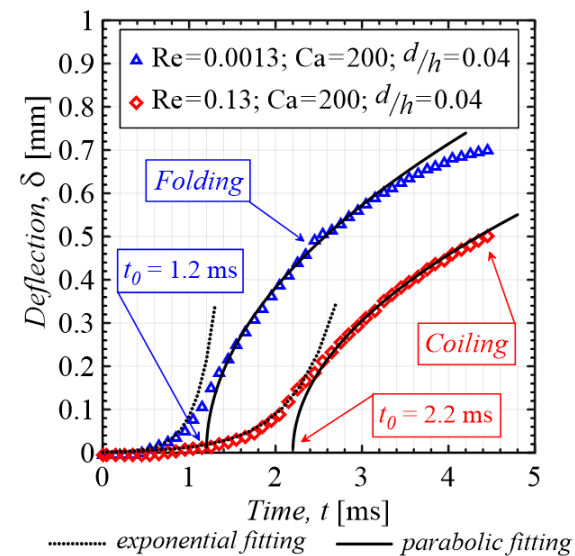
Reliable Eulerian Framework



Experimental comparisons



Extensive analysis

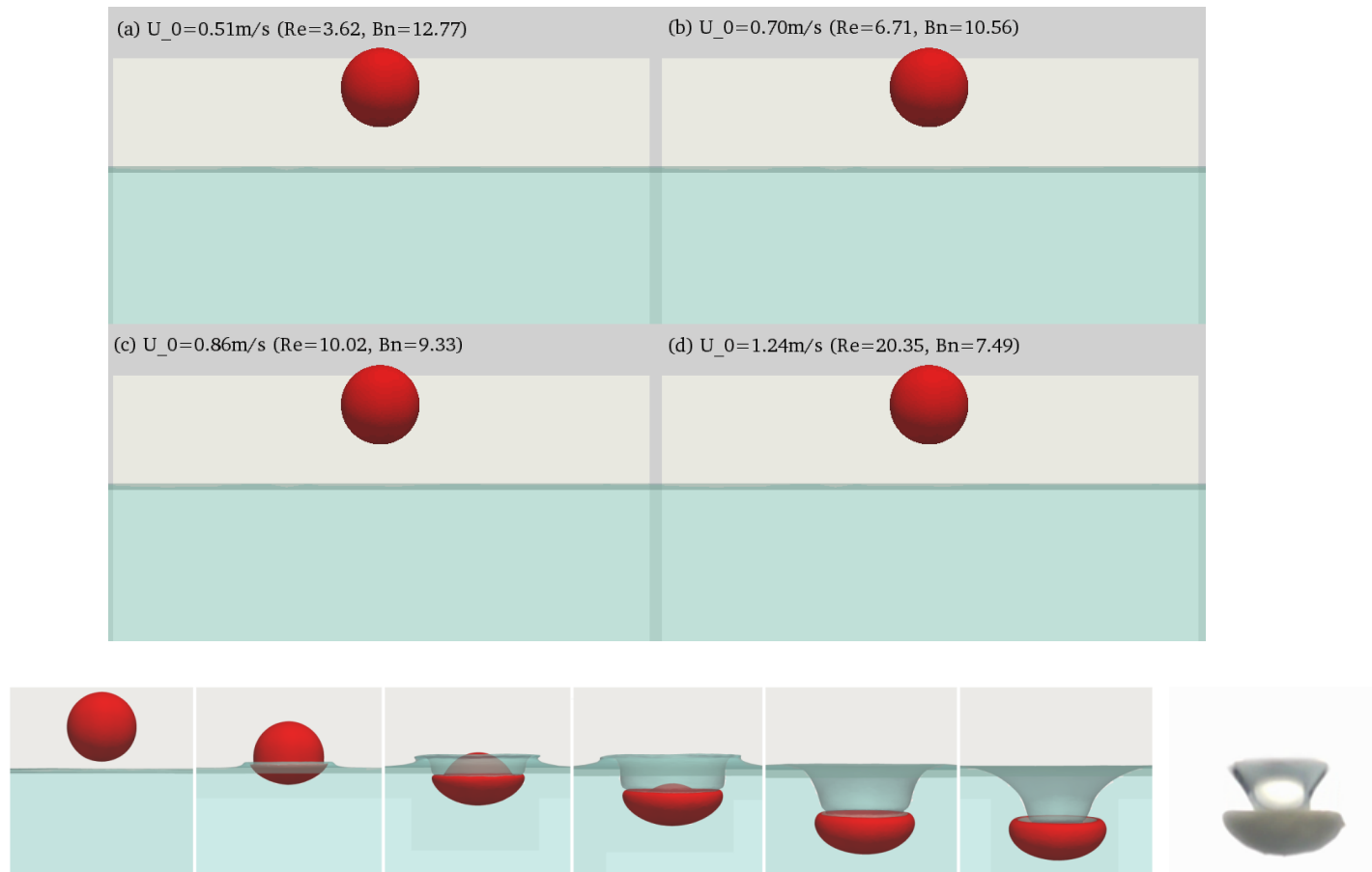


A. Pereira, A. Larcher, E. Hachem, R. Valette, Capillary, viscous, and geometrical effects on the buckling of power-law fluid filaments under compression stresses. Computers & Fluids, 190, 514-519 (2019)

## Numerical test 5: Bingham droplet in water (new benchmark)

34

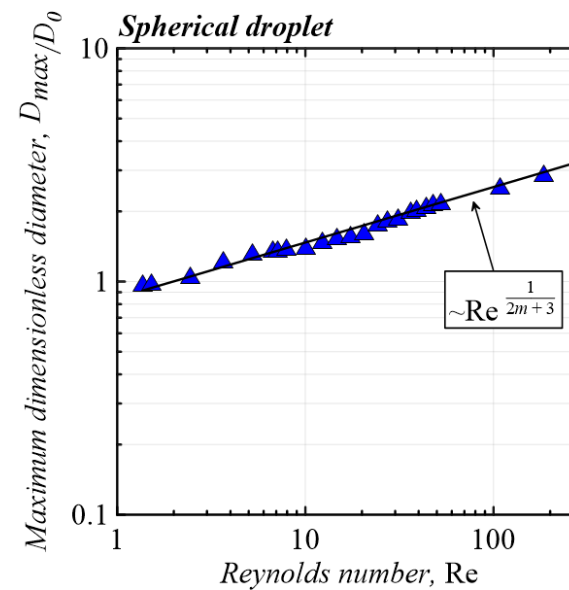
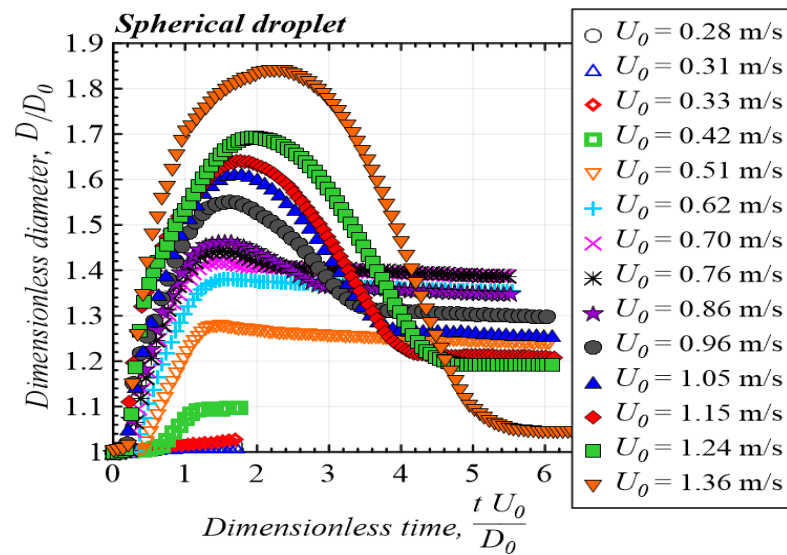
- Yield stress water-entry: “microfluidic encapsulation”



## Numerical test 5: Bingham droplet in water (new benchmark)

35

- Main results: new patent for encapsulation



## Challenge 1: phase change and boiling

36

### □ Thermal treatment in heating furnaces and **quenching tanks**

- Performed on a material at a solid state
- To alter its microstructure and properties

### □ A good thermal treatment :

- Better quality and availability of products
- Safer and secured equipments
- Reduce waste and produce durable materials
- Avoid repetitive manufacturing



### □ Manufacturing of complex components for the aerospace, aeronautics and automotive industry

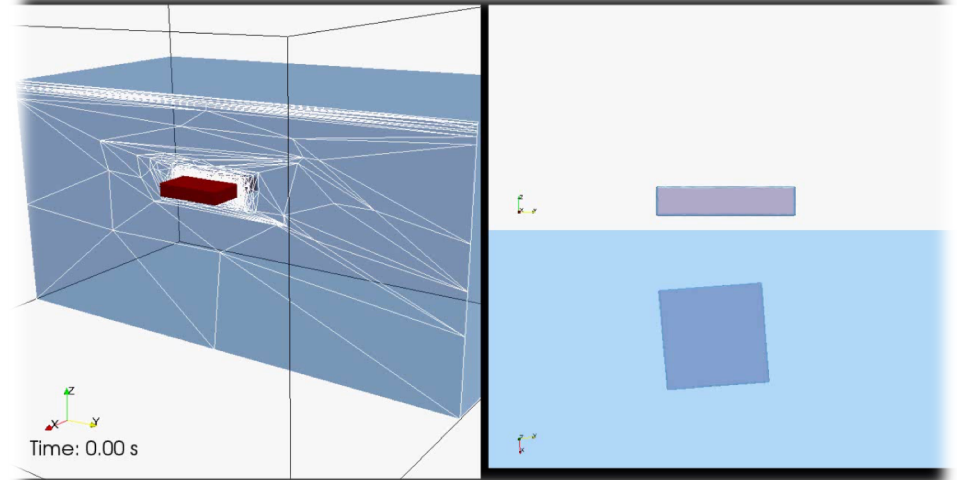
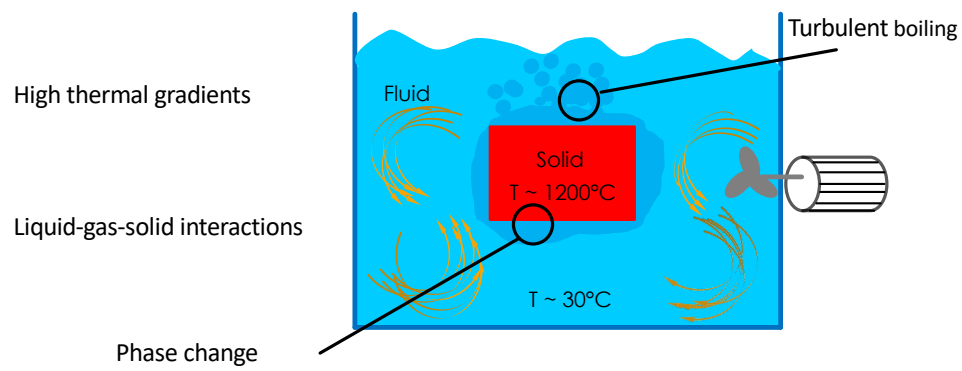




## Challenge 1: phase change and boiling

37

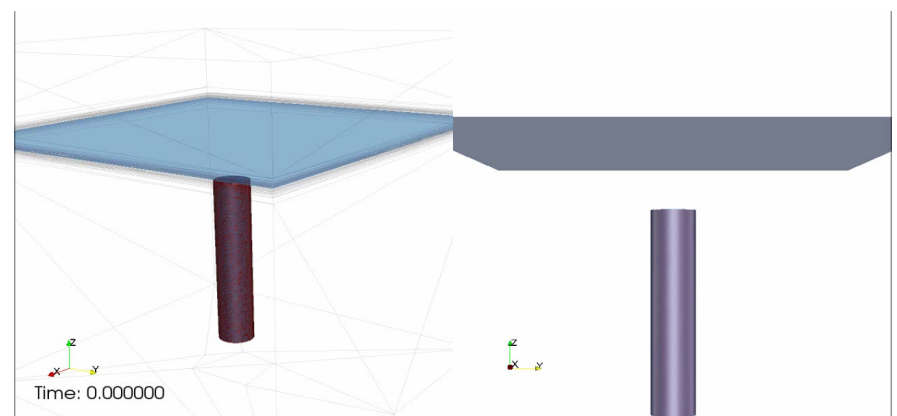
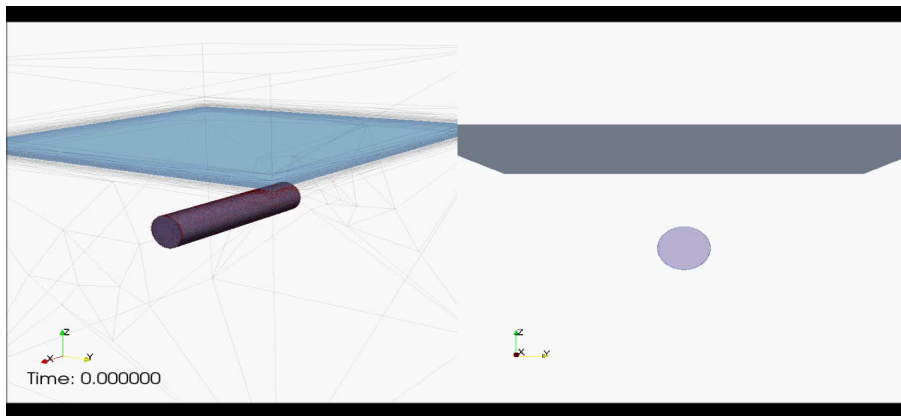
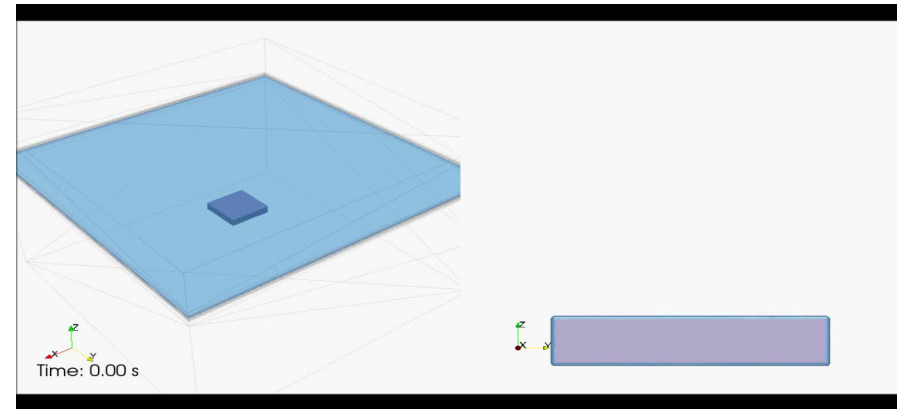
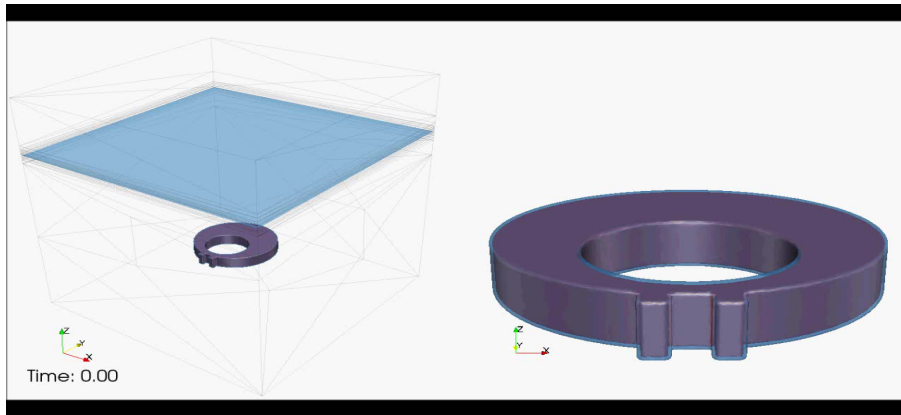
### ❑ Scheme of the quenching process



## Challenge 1: aid to decision

38

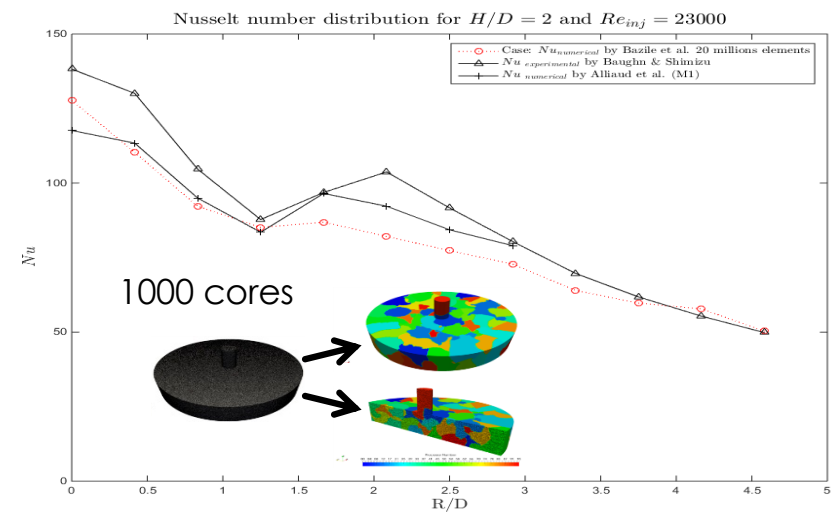
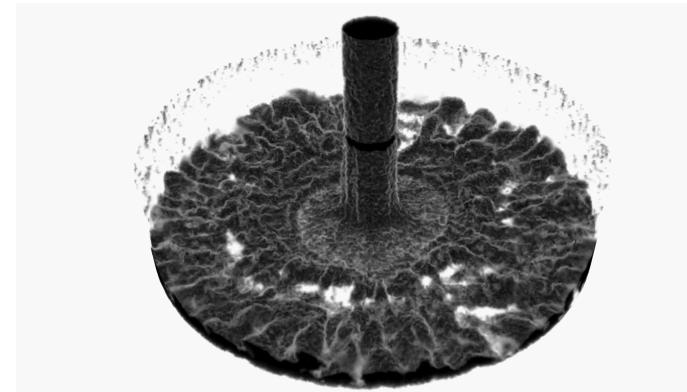
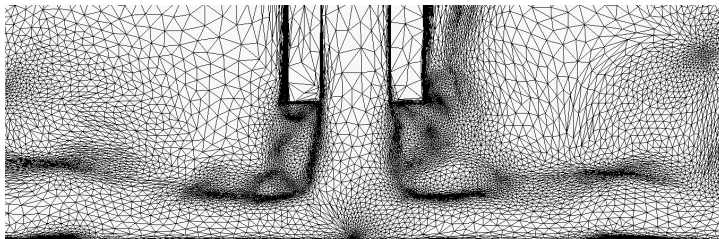
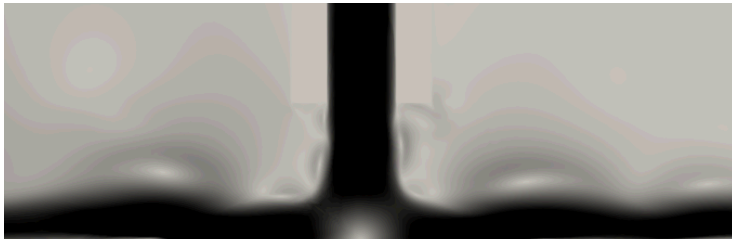
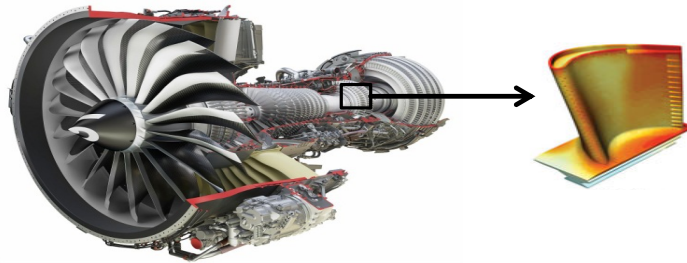
- ❑ Water tank quenching and analysis of the geometry, position, orientation...



## Challenge 2: Jet impinging for cooling

39

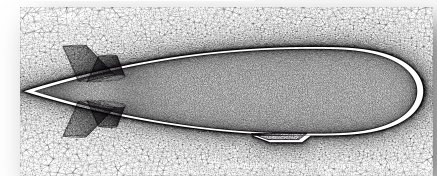
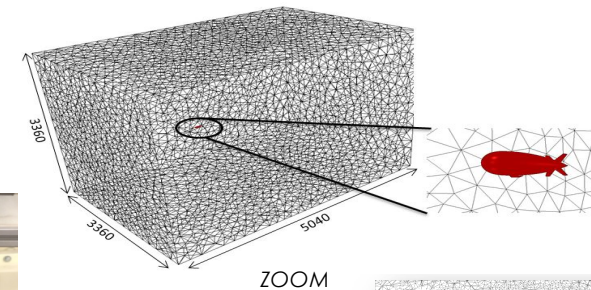
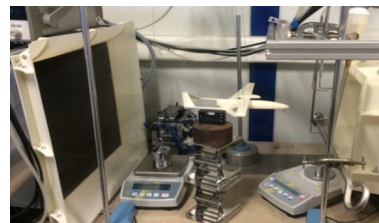
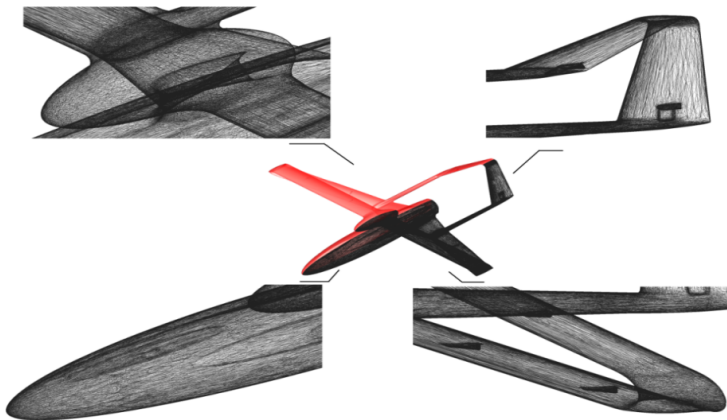
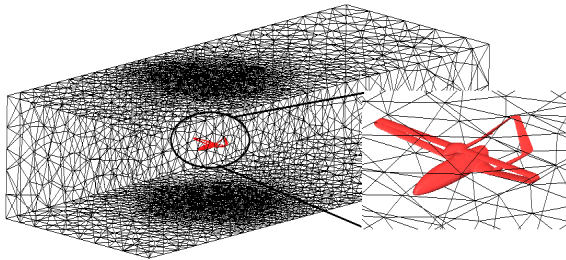
**SAFRAN**  
AIRCRAFT ENGINES



## Challenge 3: Aerodynamic performance for a drone and an airship

40

- ❑ SFEM for Spalart–Allmaras turbulence model



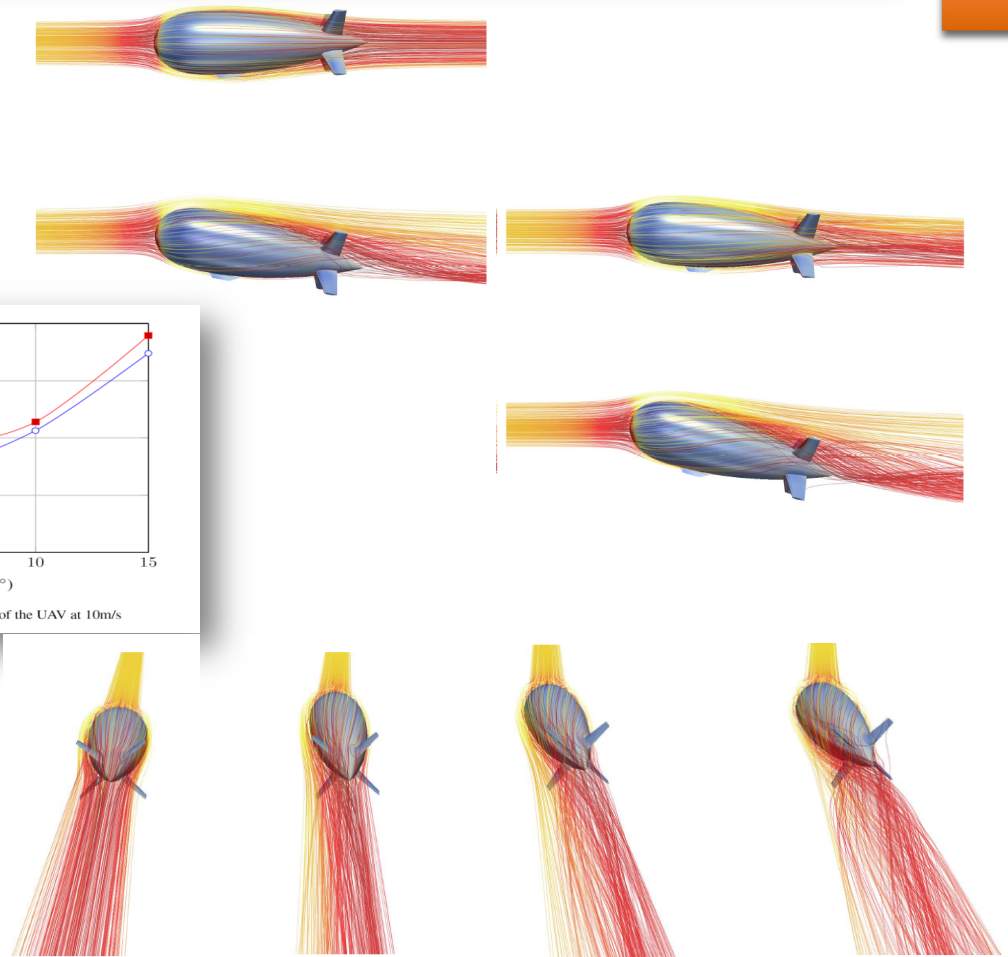
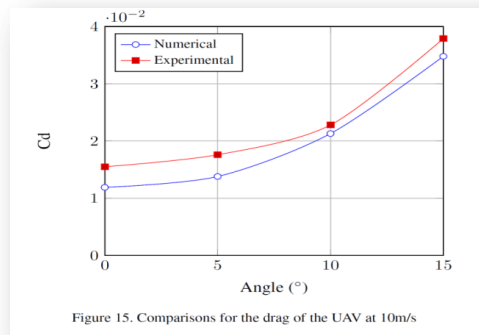
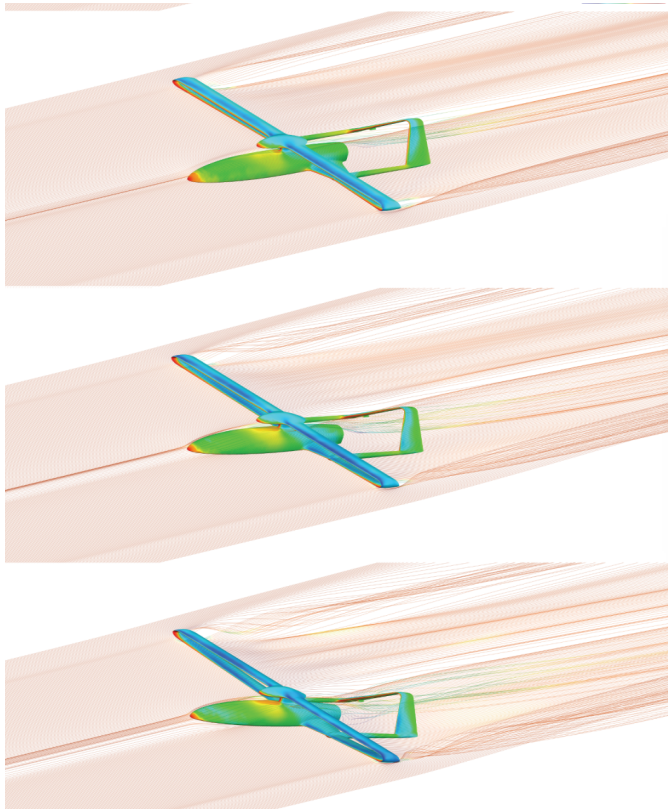
G Guiza, A Larcher, A Goetz, L Billon, P Meliga, E Hachem, Anisotropic boundary layer mesh generation for reliable 3D unsteady RANS simulations, *Finite Elements in Analysis and Design* 170, 103345, 2020

J Sari, F Cremonesi, M Khalloufi, F Cauneau, P Meliga, Y Mesri, E. Hachem, Anisotropic adaptive stabilized finite element solver for RANS models, *International Journal for Numerical Methods in Fluids* 86 (11), 717-736, 2018



## Challenge 3: Aerodynamic performance for a drone and an airship

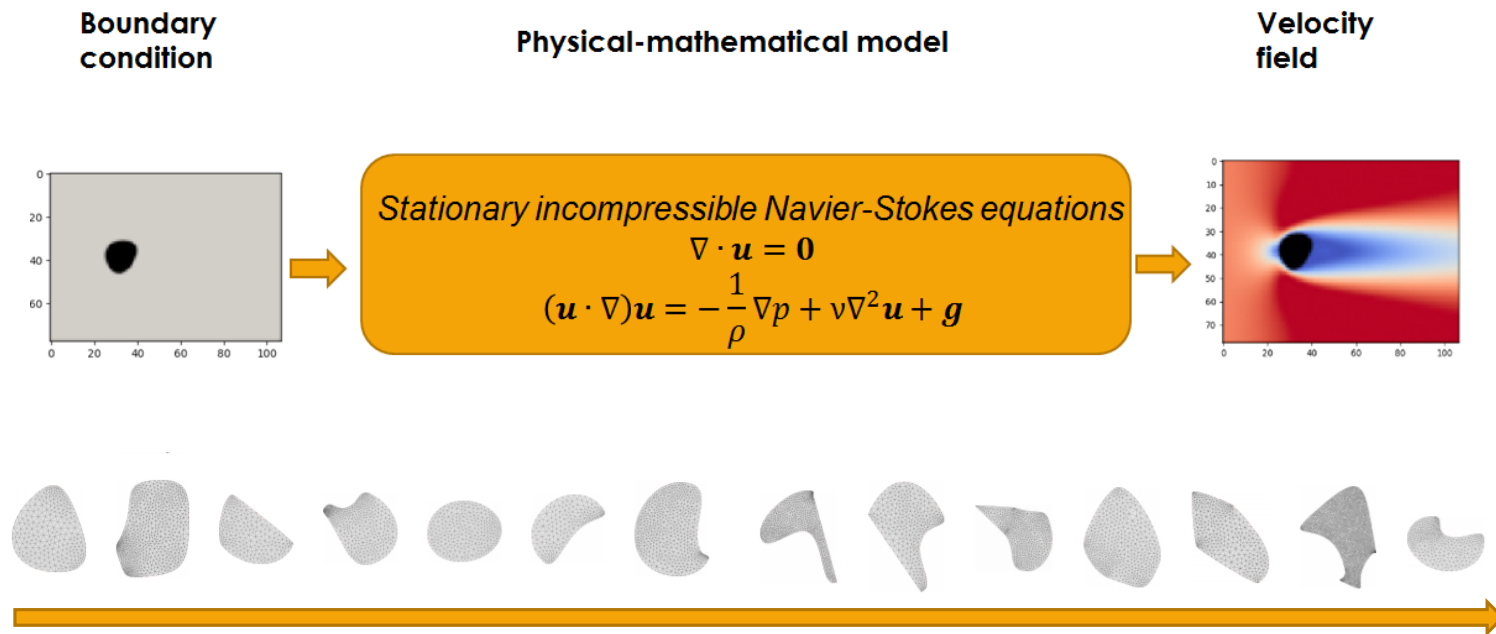
41





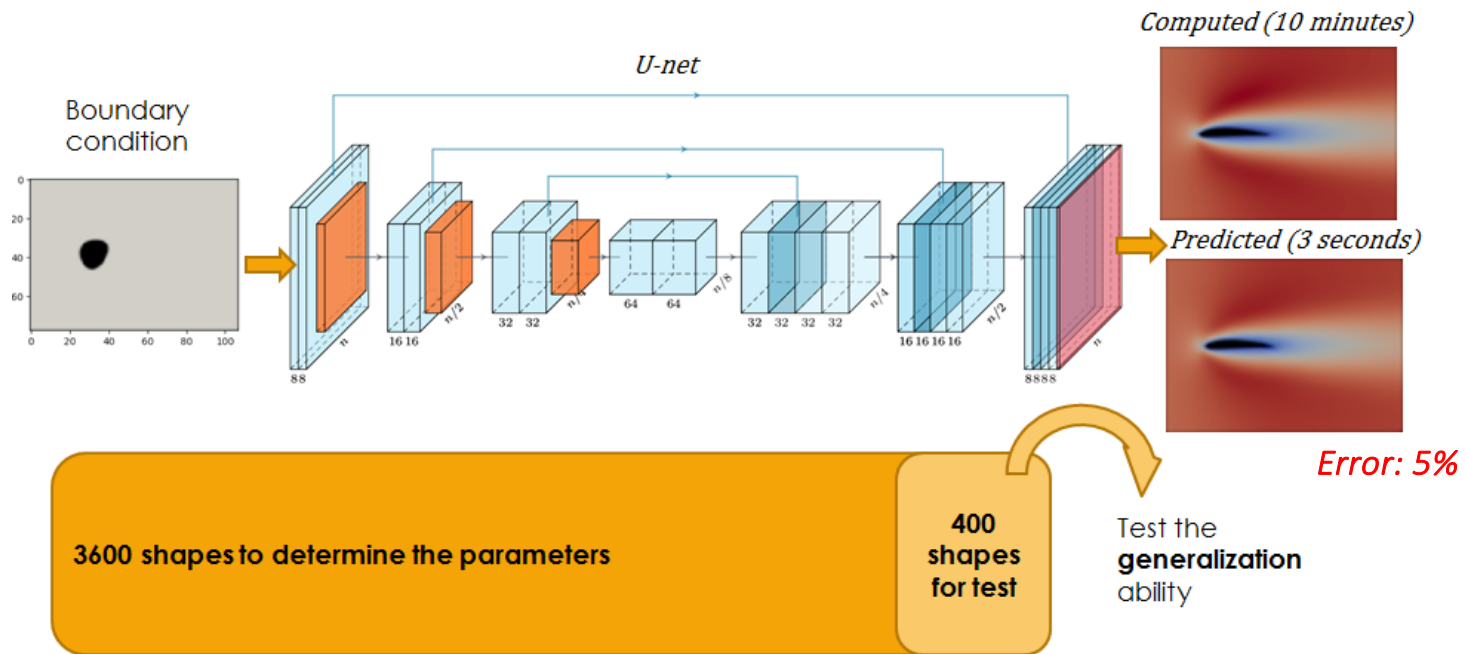
## Challenge 4: towards coupling CFD and Data Sciences

42



## Challenge 4: towards coupling CFD and Data Sciences

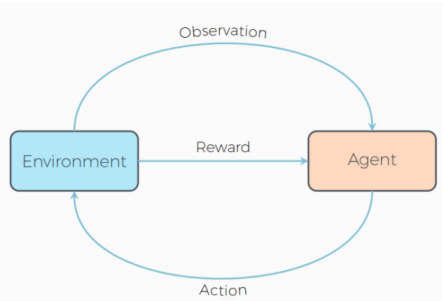
43



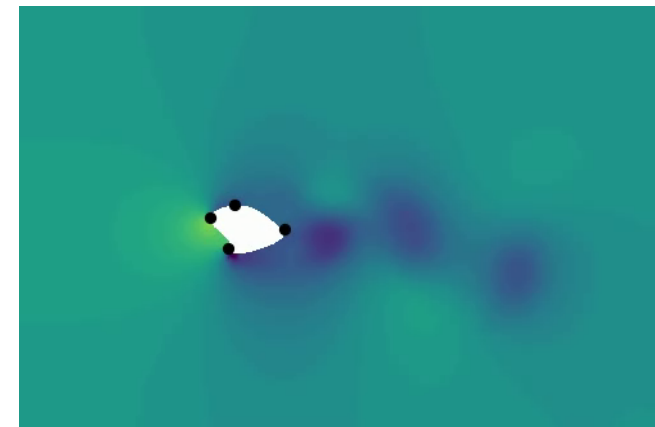
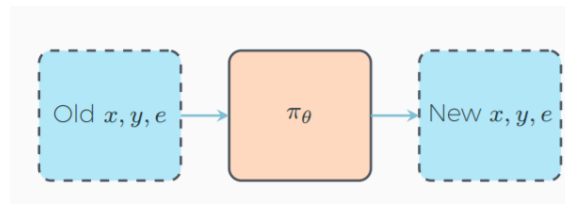
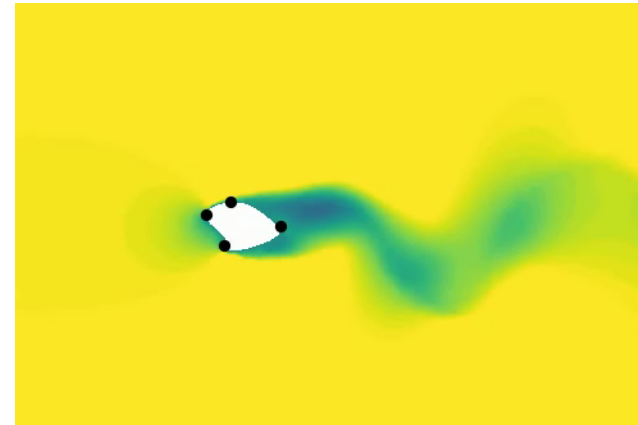
-Jonathan Viquerat, Elie Hachem, *A supervised neural network for drag prediction of arbitrary 2D shapes in low Reynolds number flows*, submitted to Computers & Fluids

## Challenge 4: towards coupling CFD and Data Sciences

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- ◇ Shape described with Bezier curves
- ◇ Channel domain, inc. N.S.
- ◇  $Re$  from 100 to 600
- ◇ 3 d.o.f. per point
- ◇ Constrained area for each point
- ◇ 2 specificities : states and short episodes

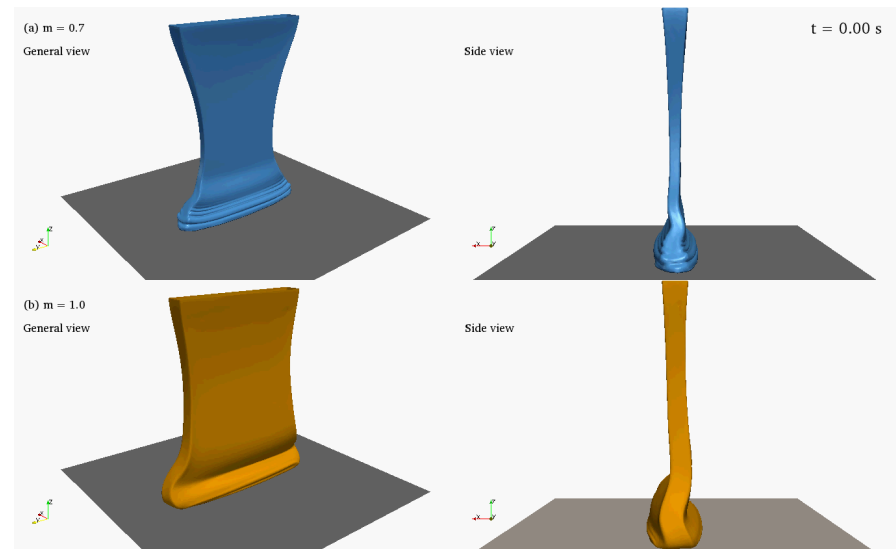
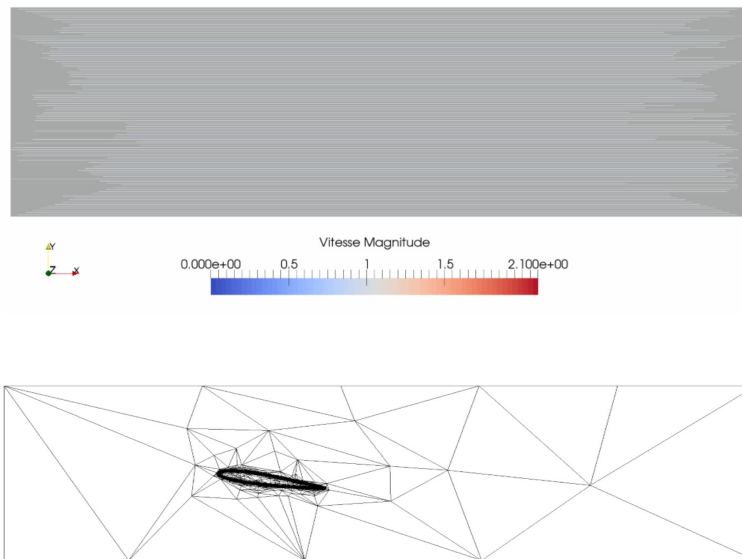


-Jonathan Viquerat, Hassan Ghroueib , Jean Rabault , Alexander Kuhnle,, Aurélien Larcher, Elie Hachem *Direct shape optimization through deep reinforcement learning*, submitted to Journal of Computational Physics

-Paul Garnier, Jonathan Viquerat, Jean Rabault , Aurélien Larcher, Alexander Kuhnle, Elie Hachem, *A review on Deep Reinforcement Learning for Fluid Mechanics*, submitted to Computers & Fluids

Thank you for your attention

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Thanks to all the members of the CFL Research group

[elie.hachem@mines-paristech.fr](mailto:elie.hachem@mines-paristech.fr)

# Conservative interpolation

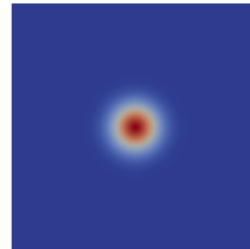
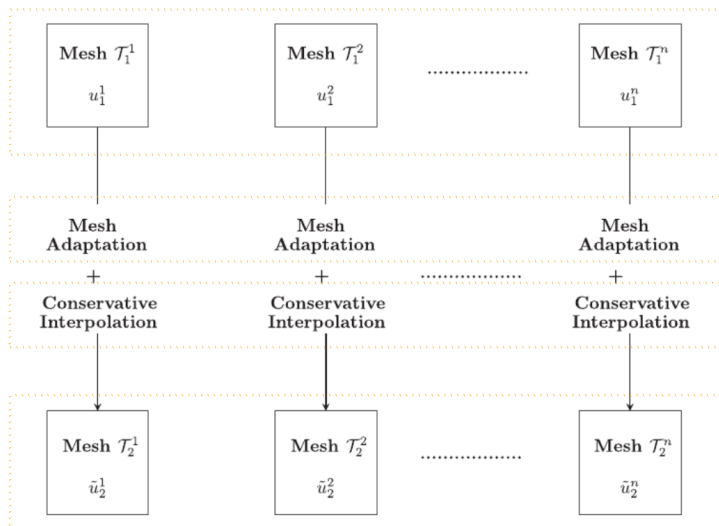
46

Ensuring conservation of linear momentum and mass

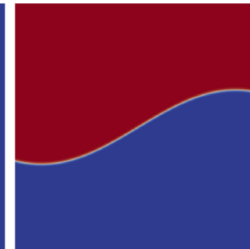
Minimize :  $\int_{\Omega_2} |\tilde{u}_2 - u_2|^2$

Under the constraints :  $\int_{\Omega_2} \tilde{u}_2 = \int_{\Omega_1} u_1$

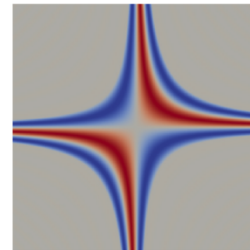
$\int_{\Omega_2} \nabla \cdot \tilde{u}_2 = \int_{\Omega_1} \nabla \cdot u_1$



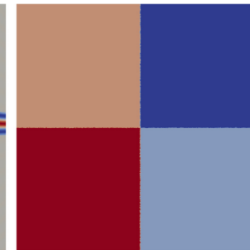
(a) Gaussian function ( $f_1$ )



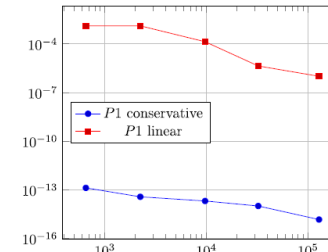
(b) Shock function ( $f_2$ )



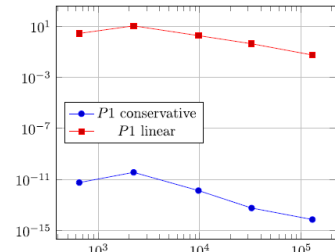
(c) Multiscale function ( $f_3$ )



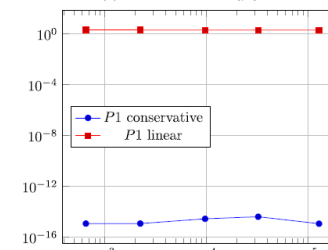
(d) Discontinuous function ( $f_4$ )



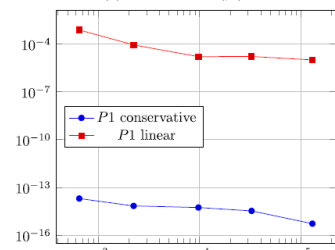
(a) Gaussian function ( $f_1$ )



(b) Shock function ( $f_2$ )



(c) Multiscale function ( $f_3$ )



(d) Discontinuous function ( $f_4$ )

3D meshes used for the different transfers.

Step	Number of nodes in $\mathcal{T}_1^i$	Number of nodes in $\mathcal{T}_2^i$
1	651	587
2	1769	1125
3	39012	29414
4	77941	71778
5	170785	113539